Eric Mercier *

Meridians of reference and mathematical geography in the Medieval Muslim West (9th-16th centuries)

Keywords: Islamic geographical coordinates, meridian of water, Ptolemy, al-Khwārizmī, longitude of the Mediterranean.

Summary: The analysis of geographical coordinates extracted from the Kennedy & Kennedy's book "Geographical coordinates of localities from Islamic sources" (1987), allows a renewed vision of mathematical geography in the medieval Muslim West (9th -16th centuries). We show that, contrary to what they themselves assert in their writings, all the ancient Arab geographers retain a "zero meridian" rejected to the west of the Canary Islands (meridian of water). This meridian was inaccessible, so that in practice they use a conventional fixed meridian located in the Middle East (most likely Baghdad and/or Damascus). We show that al-Khwārizmī (9th century) played a founding role in the creation of this system and in partial correcting the erroneous data of Ptolemy. Subsequently, especially in the 10th and 11th centuries, some authors made further global and/or local improvements, but these were soon forgotten.

Introduction

The geographical tradition of the ancient Arab-Muslim civilization revolves around two types of works. Firstly, a tradition whose founding model is the *Kitāb al-masālik wa l-mamālik* of Ibn Khurradādhbih (9th century) ("Book of Roads and Kingdoms"; or rather "Book of Roads and Realms" according to Pinto 2016 : 3) where three major concerns dominate: taxes, the situation at the borders, and roads with their distances and relays. The second seminal work is the *Kitāb ṣūrat al-arḍ* of al-Khwārizmī (9th century) ("Book of Face of the Earth") which belongs to a field that historians of Science call "Mathematical Geography" and which corresponds to the determination of geographical coordinates (longitudes and latitudes). This second part of the geographical tradition, to which we will focus here, is directly inspired by Ptolemy (c. 90 - c. 168), the famous scholar of late antiquity. The preoccupations that guide works of this type are clearly oriented towards cartography[1].

Ptolemy had established a mathematical planetary geography system defined by angular coordinates (latitudes and longitudes), counted respectively from the equator and from the meridian of the Canary Islands (Fortunate Islands). The Arab geographers who appropriated this system from the beginning of the 9th century, very quickly realized the limits of this system. The main shortcoming was that the "prime meridian" was not easily accessible, and this posed problem if they wanted to set astronomical measurements or simply distance measurements. To solve this problem, some authors have chosen to refer to other meridians. Thus, for example, al-Marrākushī (Sédillot 1834 : 312-13) explains that he based his calculations of longitude on the meridian that passes through the middle of the known world

* Professor, Université de Nantes, Faculté des Sciences, UMR du CNRS 6112, Laboratoire Planétologie et Géodynamique, 2 rue de la Houssinière, BP 92208, 44322 Nantes cédex 3 –France, eric.mercier@univ-nantes.fr

[1] The most relevant study of the relationship between mathematical geography and medieval Arab-Muslim cartography corresponds to the first chapter of Rapoport (2020: 11-39). More generally, this book is the best available introduction to Islamic cartography.

("Dome of Arîn", "Qubbat Arīn" or "Dome of the Earth"), even though he expresses his results using the same "zero meridian" as Ptolemy. This example illustrates well the disconnection that exists, in Muslim mathematical geography, between the "zero meridian" (here: Canary Islands) and the "reference meridian" (here: Dome of Arîn). Generally, these notions are confused in the term "prime meridian" that I will avoid from now on.

On the other hand, other geographers have completely abandoned the Ptolemy's zero meridian. Indeed, as formalized by Kennedy & Regier (1985), the Arabic geographical tables that have come down to us can be classified into two systems (A and C) of almost equivalent numerical importance. While group C continues to refer to Ptolemy and the meridian of the Canary Islands (C for Canary Islands), group A refers, explicitly, to a meridian on the Atlantic coast of Africa located 10° further east (A for Atlantic); the longitudes of the two groups being offset by 10° (see also on this subject: Tibbets 1992 : 103, Kennedy 1996 : 187, Sezgin 2005 vol.1 : 149, King 2014 : 27).

Furthermore, and as shown by Comès (1994, 2000), some coordinate tables are based on a zero meridian shifted, but this time towards the West, with respect to the position fixed by Ptolemy. This is the principle of the "water meridian"[2] which, according to this author, would have been adopted only in the Muslim West. As the gap has varied over time, there are several water meridians.

The existence of these different meridians (Canary Islands, Atlantic Coast, Dome of Arîn, the different meridians of water) generates extreme confusion, a confusion that is unanimously underlined by the authors of the syntheses on ancient Arabo-Muslim Mathematical Geography (Tibbets 1992, Kennedy 1996, Sezgin 2005). In the following lines, I propose to revisit this problem by taking a historical approach. I plan to show, step by step, how these different meridians were introduced, and to discuss whether they had an operational role or whether, on the contrary, they were merely declarations of principle without practical consequences. For this I will use a database of more than 12,000 ancient coordinate pairs, which was compiled by Kennedy and Kennedy (1987) and which has been only rarely and only very partially used by cartographic historians[3]. In the course of this exploration I will be led to highlight a serious misinterpretation, or negligence, of the Arab geographers who have left us writings. This error was later taken up by almost all historians, including very recent ones; I will show why, in my opinion, this error has caused the image of confusion that is the one mentioned by recent authors. Once the problem of the reference meridian has been solved, I will show that it is possible to have an overall vision, totally renewed, of the evolution of ancient Mathematical Geography in the western part of the Muslim World. I will develop two examples, one global and one more regional.

The database used

In 1987, Edward S. and Mary H. Kennedy published a compilation of over 12,000 pairs of geographic coordinates from 76 different Islamic sources[4]. These are "Geographical coordinates of localites from

[2] This name did not appear explicitly until 1321 in a Western book ("Portuguese Almanac of Madrid"), but it is indeed the same concept that it is about (Comes 1994).

[3] Regier (1987), Robles Macías (2014).

[4] to which are added modern values (Greenwich) and those of Ptolemy for comparison.

Ain-i-Ākbari of Abul Fazl-i-ʿĀllami	AIN	c. 1580	C	656
Ulugh Beg's Zīj-i Sulṭānī	ULG	c. 1440	C	278
Al-Kāshī's Zīj-i Khāqānī	KAS	c. 1420	C	515
An anonymous table in Gotha MS 1467	GT2	before 1467	C	59
Ibn al-Shāṭir's al-Zīj al-jadīd	SHA	c. 1350	A	251
The Zīj of Shams al-Munajjim	MUN	c. 1330	C	256
The Ashrafī Zīj of Sayf-i Munajjim	ASH	1310	A	244
An anonymous Zīj rreported by Abūl-Fida'	ZDJ-FID	before 1310	A	31
The qiyâs reported by Abūl-Fida'	QYS-FID		A	94
ibn al-Bannāʾ 's Minhāj al-ṭālib	BAN	before 1321	A	32
The Zīj of Jamāl ad-Dīn al-Baghdādi	BAG	c. 1285	A	157
al-Marrākushī's Jāmiʿ al-mabādiʾ	MAR	c. 1250	C	134
A Zīj by Muḥyī al-Dīn al-Maghribī	MAG	1276	C	211
A table in Leiden Ms.Cod. 1001(15)Warn.	LYD		A	170
Nasîr ad-Dîn al-Tûsî Zīj-i Īlkhānī	TUS	c. 1270	C	272
Tiij al-azyāj by Mulyī al-Dïn al-Maghribī	TAJ	1258	C	146
Al-Tūqānī's Risālafi'l-amal bil-asturlab	TUQ		C	151
ibn Saʿīd as reported by Abūl-Fida'	SAA-FID	before 1286	A	210
ibn Saʿīd's Kitāb bast al-ard	SAA	before 1286	A	432
A Zīj reported in Yāqūt's Mu'jam	ZIJ-YAQ	before 1220	A	32
The anonymous Marseilles tables	MRS	12th cent.	A	55
The list in the Toledan Tables	TOL	c. 1185	A	65
The Sanjarī Zīj, Istanbul Copy	SNH	c. 1120	A	111
The Sanjarī Zīj, British Museum Copy	SNB	c. 1120	A	162
al-Zayyāt's Kitāb al-aqālīm	ZAY	before 1058	A	298
Bīrūnī's Qānūn reported by Abūl-Fida'	BIR-FID	c.1040	A	279
al-Bīrūnī's Al-Qānūn al-Masʿūdi	BIR	c. 1040	A	604
ibn Yūnus's Al-Zij al-Kabîr al-Hâkimi	YUN	c. 990	A	291
al-Battānī's aAl-Zij al-Sabi'	BAT	c. 900	C	273
The anonymous Kitāb al-Atwal	ATH-FID		A	452
ʻajāʾib al-aqālīm al-sabʻah by Suhrāb	SUH	c. 930	A	492
Kitāb Rasm al-Ma'mūr, from Abūl-Fida'	RES-FID	c. 820	A	91
The list attributed to al-Khuwārizmī	KHU	c. 820	A	545
The Geography of Ptolemy	PTO	c. 140	C	213
The Handy Tables of Ptolemy	HTP	c. 140	C	362

Table 1: Chronological list of sources used (35 of the 76 sources in the K&K compilation); Computer Code (4 or 7 signs), datation and number of localities. The list order and dates are taken from K&K, which will be consulted for further details.

Islamic sources," which I will refer to as "K&K" in this article. These sources are either manuscripts or modern works reproducing ancient geographical tables. Each source, which is assigned a 3- or 7-characters code to be used in this article (see Table 1), contains the coordinates, longitude and lati-

tude, of a variable number of localities (2 to 656). K&K have proposed an A or C allocation for the vast majority of sources and an overall chronological ranking.

As most of the sources are dated, a solid chronological framework is available to study the evolution of ancient Arabic mathematical geography. In this study, the following will be excluded:

- sources from the 17th century onwards, possibly influenced by western geography,
- sources mentioning only oriental localities,
- sources that appear problematic or highly corrupted and for which K&K has not determined membership of groups A or C,
- the QBL source, for which there is a major dating problem[5].

This means that 35 sources (46%) will be used, out of the 76 sources collected by K&K (table 1). This represents more than 8620 pairs of coordinates..

Ptolemy, his errors, and the corrections of al-Khwārizmī (9th century)

Figure 1 illustrates the errors made by Ptolemy on latitudes and longitudes measured from his zero meridian (Canary Islands meridian). It can be seen that the errors on latitudes are not correlated with longitude. It is also noted that they are quite small (90% of the errors are less than 0.89° or about 100 km; n=188). This is not surprising because the methods of measuring latitude are simple and have been mastered since antiquity. The errors of longitudes are much more significant, again this is not surprising since, at the time we are interested in, geographers had only uncertain and approximate methods[6] to make this measurement. Moving eastward, the mean error increases linearly.

The first type of longitude error, which can be called "geodetic", is due to an underestimation of the Earth's diameter by Ptolemy (e.g. Evans 1998 : 65). It corresponds, for example, to an overestimation of the longitudinal distance between the Strait of Gibraltar and the Middle East by about 20°. In addition to this geodetic error, there are errors of detail in figure 1 that can be described as residual[7]. Arab geographers who inherited the Ptolemy system therefore had two types of error to correct. The first medieval Arab author who worked to improve Ptolemy's geography is al-Khwārizmī, in a book known in one copy[8] and edited by von Mzik (1926) and commented by him and Nallino (1939): *Kitāb Sūrat al-ard*. In this work, directly inspired by Ptolemy, al-Khwārizmī corrects both the lati-

[5] The Kitāb corresponding to the QBL source is the only known manuscript from Zayn al-Dīn al-Dimyāṭī, of whom we know very little. This source is not dated, but K&K places it chronologically between YUM and ZAY (late 10th century- middle 11th century). According to King (1984 : 115 : footnote 70) Zayn al-Dīn al-Dimyāṭī is a 12th century Egyptian author. For her part, Comes (1994 : 48) considers that he lived in the 17th century.

[6] These methods are: (1) The dead reckoning : knowing the size of the Earth and the orientation and length of the straight line joining two localities, we can calculate the differences in coordinates and in particular the longitude component. This is obviously a very imprecise method because of the difficulty of accurately estimating distances and azimuths ashore; (2) The simultaneous observation and timing of cosmic events, including lunar eclipses, from two locations on Earth: a such observations makes it possible to evaluate the time difference and therefore the longitude component of the distance between the two locations. But these are rare cosmic circumstances and, in addition, they must be planned in advance in order to schedule astronomical observations. This method is mentioned by many medieval Muslim authors, but it was already known at the time of Hipparchus (2nd century BC) (see Comes 2000 for a complete review).

[7] That is, the error remaining after removing the geodetic error.

[8] University of Strasbourg (MS 4-247), available on “gallica.bnf.fr”

tudes[9] (Fig. 2B), and the longitudes (Fig. 2A), provided by the latter. As far as the longitudes are concerned, there is a slight increase, significant because systematic, in the longitudes of the Maghreb (about 1.5°) and a decrease from an average of 8° to 8.5° in the longitudes of the Middle East. This amounts to a decrease of about 9.5° to 10° in the longitudinal distance between these two regions[10]. About half of the Ptolemy geodetic error is thus corrected.

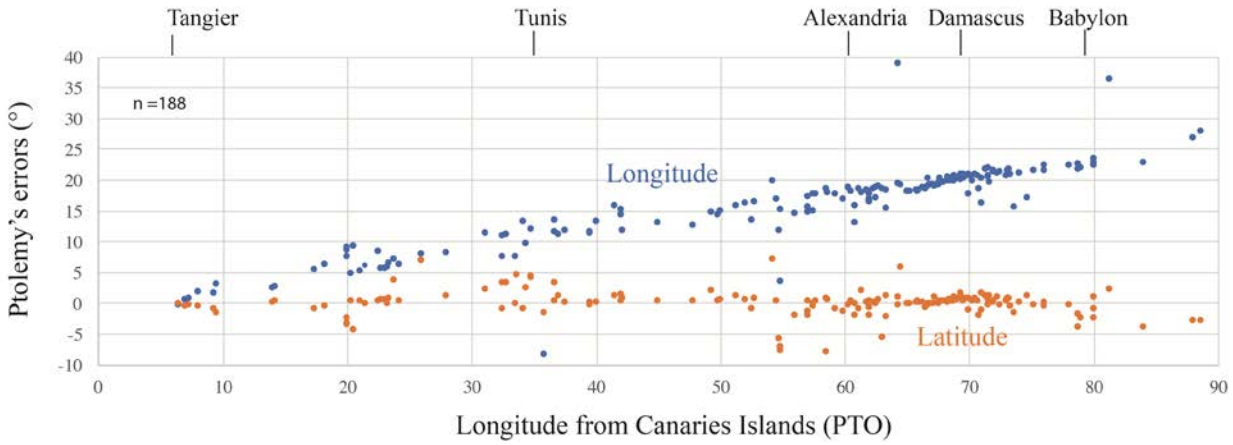

Figure 1: Errors (with respect to current data) made by Ptolemy on the latitude and longitudinal distance to the meridian of the Canary Islands. It can be seen that the further east the locality considered, the greater the error of longitude (data according to K&K). Thus, for example, the longitudinal length of the Mediterranean is overestimated by about 20 degrees..

It is certain that al-Khwārizmī did not have at his disposal a new astronomical determination of the longitudinal distance between the Canary Islands and any locality in the Western Maghreb. The average increase of 1.5° of the longitudes in this region can only correspond to a shift of the zero meridian. However, Al-Khwārizmī is considered (K&K, Kennedy 1996) to be the first to use system A which, it should be recalled, is defined by a zero meridian on the Atlantic coast, 10° east of the zero meridian of Ptolemy (Fig. 3B). If it was this meridian that al-Khwārizmī had used, the longitudes of localities in the western Maghreb would have to be reduced by 10°; but this is not the case, on the contrary, there is an increase of 1.5° on average. It is therefore clear that al-Khwārizmī refers neither to the meridian of the Canary Islands (Fig. 3A) nor to the meridian of the Atlantic coast (Fig. 3B), but to a meridian further west (Fig. 3C). This first conclusion, which had already been partly drawn by Nallino (1939), but has since been forgotten, is very important and we will have the opportunity to come back to it several times.

[9] 90% of the latitude errors of KHU, compared to modern data, are less than 1.35° (n=220), indicating an accuracy on this coordinate less than PTO. Compared to Ptolemy (PTO) his corrections are, at 90%, less than 0.76° (n=159).
[10] Recent authors have often used the "Longitudinal Length of the Mediterranean" or "Longitude of the Mediterranean" as a proxy for this distance (Robles Macías 2014).

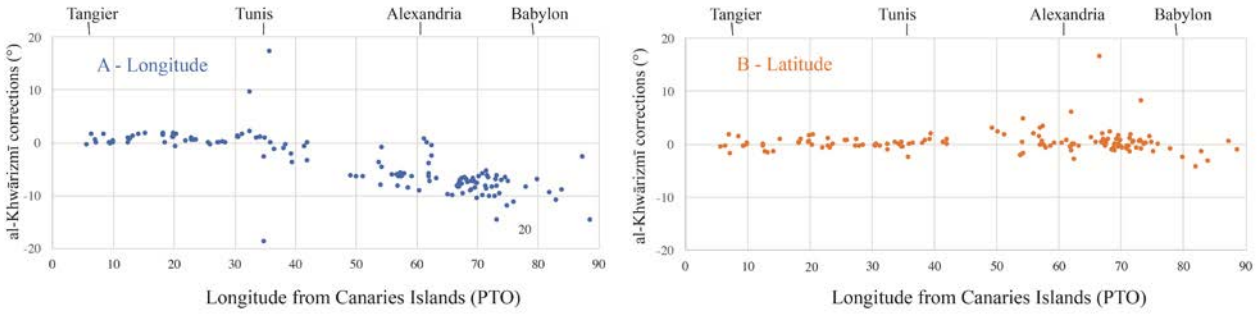

Figure 2: A: Correction, by al-Khwārizmī (KHU), of the longitudes (A) of Ptolemy (PTO),(i.e.: longitudes of KHU - longitudes of PTO). B: The same for latitudes.

If al-Khwārizmī does not use the Ptolemy meridian or the Atlantic coast meridian as a reference, one must wonder which other fixed meridian he has chosen. In other words, what is the Ptolemy meridian that he keeps and on which he bases his calculations of longitude. The simplest hypothesis would be that he refers to the 90° meridian of Ptolemy: that of Qubbat Arīn (hypothesis of Kalin et al 2014 : 368, for example). The problem is that this meridian is very poorly defined and in his work he does not seem to try to improve this situation; he does not even seem to allude to it (von Mzik 1926, Nallino 1939). Moreover, in his table he mentions two localities at the longitude of 90° (Kerman, in present-day Iran, and Muhamadiyya, probably Ray in present-day Iran). The first is the only one mentioned by Ptolemy, but at the longitude of 100°. Clearly the 90° meridian of al-Khwārizmī does not correspond to the 90° meridian of Ptolemy.

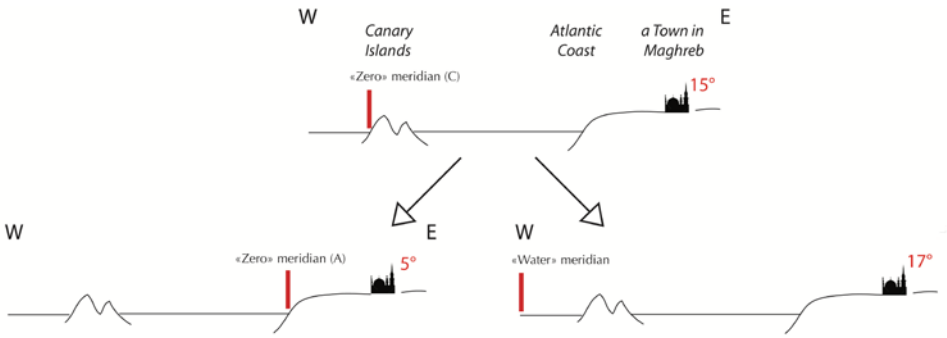

Figure 3: Schematic diagram showing that an increase in the longitudes of Maghreb localities is the corollary of a westward shift of the "zero" meridian; conversely, if the "zero" meridian is located on the Atlantic coast, one should observe a decrease in the longitudes of these same localities, which is not the case for any of the sources studied.

We must therefore take another hypothesis Alas, al-Khwārizmī does not explain his method anywhere; it must be deduced from his table. I assume that al-Khwārizmī chose a reference meridian in an important city in the centre of the Empire. If he had wanted to keep the Ptolemaic value at this meridian, he would have had to move the zero meridian from 9° to 11° to the west, depending on the locality chosen. But this is not what he did. The hypothesis that I propose and that he has chosen to assign to his reference meridian in the Middle East, a correction corresponding to a "round number":

10°, and recalculate all the longitudes from this reference. If we accept this hypothesis and if we look for a major Middle Eastern city whose longitude is shifted by 10° between PTO and KHU, we only find Jerusalem (PTO: 66.0°; KHU: 56.0°). This al-Khwārizmī's choice imposed that the zero meridian undergoes a "small" shift of 1.5° westward from the zero meridian, but this did not present major disadvantages as this meridian was inaccessible anyway.

Tables from 10th to 13th century

Among the sources compiled by K&K (Table 1), two groups can be distinguished. The first corresponds to sources that largely follow the ideas of al-Khwārizmī; the second group, which originates from the table of al-Battānī (BAT; early 10th century), does not use the same basic postulates, but, as we shall see later, is characterised by a parallel evolution to that of the first group.
In practice, the division between these two groups corresponds precisely to that proposed by Kennedy & Regier (1985) or K&K (systems A and C). But, as we shall see, it appears that these authors were mistaken about the zero meridians used in these two groups.

The followers of al-Khwārizmī (Group A)

In K&K we find a series of sources that present strong analogies with the geographical table of al-Khwārizmī (KHU). These sources are all characterized by (1) a Middle East with longitudes reduced by 8 to 10° with respect to PTO, and (2) in the Western Maghreb, longitudes higher than those of PTO. This increase in longitudes in the western Maghreb can be quite variable: between 1 and 2° (Fig. 3, ex. SUH), we have values close to that of KHU, or higher without generally exceeding 12° (example Fig. 4, ex: ZAY), I will come back to these differences.

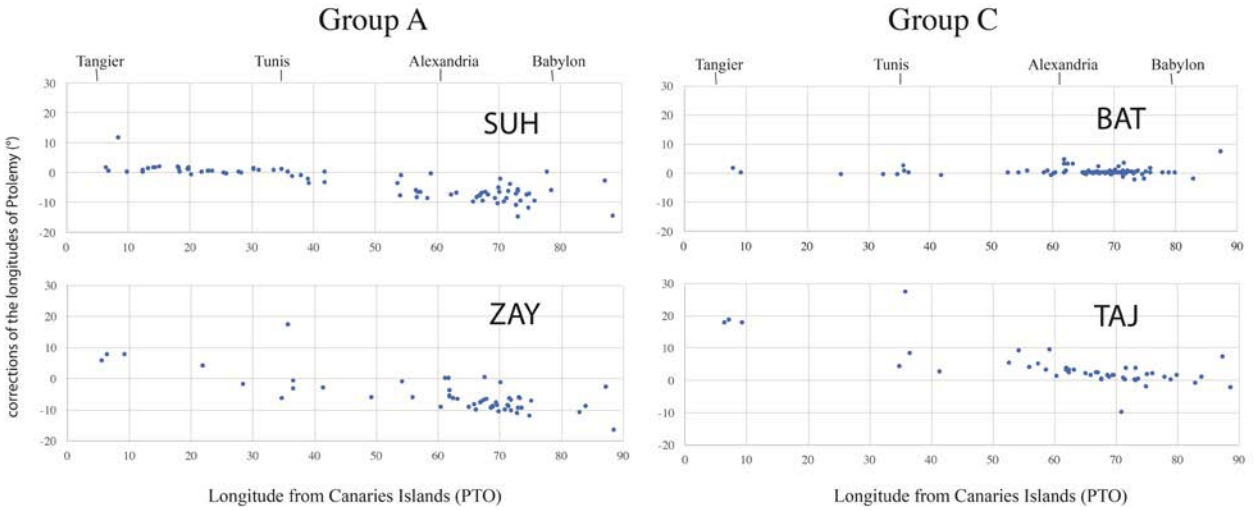

Figure 4 : Examples of corrections of Ptolemy longitudes (PTO) by different sources (SUH, ZAY, BAT and TAJ). It can be seen that SUH corrections are close to those of KHU (Maghreb: about +1.5° and Middle East: about -8.5°) (see Fig. 2). ZAY is identical for the Middle East, but applies a larger correction to the Maghreb (+8°). BAT corrects very little for PTO longitudes. This is also the case for TAJ for the Middle East, but the correction for the Maghreb is large (+18°).

As in these sources, the longitudes of the Western Maghreb are greater than those of Ptolemy, because the zero meridian is west of the Canary Islands (water meridian; Fig. 3). Never in this group, which corresponds precisely to group A of Kennedy and Regier (1985), can we identify a zero meridian located east of the Canary Islands and a fortiori on the Atlantic coast.

That said, it is certain that the authors of the tables themselves did not pay any particular attention to the real position of their zero meridian, to the point of committing gross errors in their writings.

- Thus, for example, al-Zayyāt (10th-11th century) (ZAY), which is one of the sources that refer most to the West the meridian zero (see Comes 1994, Mercier in press), defined, in his *Ḏikr al-Aqālīm*, the meridian zero as did Ptolemy (Castelló 1989 : 56 & 88).
- Al-Bīrūnī (11th century) (BIR) explains in his *Kitāb Taḥdid nihāyāt al-amākin li-taṣḥīḥ masāfāt al-masākin* (translation: Ali 1973 : 121) why Arab geographers historically use a zero meridian shifted 10° to the east from that of Ptolemy. It is this position that he claims to use[11]. According to Sezgin (2005 : 149), al-Bīrūnī, in his *Qānūn*, reports the same 10° eastward shift of the zero meridian. But this assertion is not compatible with the longitudes he displays for the Muslim West; these are indeed very close to those of al-Khwārizmī. In the same vein, it should be pointed out that it indicates a longitude of 100.8° for the Dome of Earth: (K&K : 196), which obviously makes no sense! In fact, in his writings, he is much more interested in the relative longitudinal distance between localities, an area in which his contribution is fundamental, than in absolute measurements and the formal definition of reference meridians.
- Finally, last example, Abūl-Fida' (XIII-XIVth century) who reports, in his *Taqwīm al-buldān*, at least 4 sources mentioned here, envisages only two possible zero meridians, the Canary Islands and the Atlantic coast, and says he refers exclusively to the second (translation Reinaud and Guyard 1848-1883: vol. IIa : 9), which is obviously not the case with regard to the longitudes he indicates.

Finally, it should be noted that the authors of the sources in this group did not remain faithful to Jerusalem as a fixed meridian. One could think that Muslim geographers chose to refer to the longitude of Baghdad because it was the capital of the Empire, but the Damascus meridian is also a good candidate, this point will be discussed after considering the sources of the group C.

Al-Battānī and his followers (Group C)

Al- Battānī (BAT), about eighty years after Khwārizmī, does not seem to be familiar with the latter's writings and will create a second tradition with his *Kitāb az-zīğ aṣ-ṣābi'* (translation and commentary: Nallino 1899-1907). In this book, he provides a geographical table of 273 localities with longitudes very close to those of Ptolemy (Fig. 4, BAT). The reference meridian and the longitude of the Mediterranean are those of Ptolemy; it is therefore a regression with regard to geographical accuracy. But given the prestige of al-Battānī, this work has had the effect of "rehabilitating", in a way, the Ptolemaic longitudes in the Middle East. Subsequently several sources will take up this conception: the first,

[11] Al-Bīrūnī claims that scholars from the eastern part of the Empire have increased the size of the Known World, and have evaluated it at 190° (or 193.5°). This is, according to him, what would have led to the shift of 10° towards the East zero meridian of Ptolemy by these authors. While confirming the position of the zero meridian of Al-Bīrūnī, Kennedy (1973 p. 91) vigorously contests his historical explanation.

according to K&K's chronology, is QBL, but there are such problems of dating on this source[12] that it is better not to take it into account in this historical analysis. Second, there are eastern sources[13] that do not provide longitudes for the Maghreb, so it is not possible to know the zero meridian implicit in these sources.

It is only from the 13th century onwards that sources from this group C appear, mentioning western localities (TUQ, TAJ, TUS, MAG, MAR, MUN, GT2, KAS, ULG and AIN). Their analysis shows that they introduce shortening of the Mediterranean Sea from 11 to 18° (Mercier in press) and a return, of equivalent value, of the zero meridian to the west of the Canary Islands (water meridian). It should be pointed out that these sources, which are in the tradition of al-Battānī (group C), have all admitted a reduction in the size of the Mediterranean, following *de facto* the conceptions of al-Khwārizmī.

But as in the previous group, it appears that the authors are not aware of referring to a zero meridian different from that of Ptolemy.

> -For example, al-Marrākushī (MAR) at the end of the 13th century, states that there are only two possible meridians of reference, either the Canary Islands or the Dome of Arîn. In this second case, the longitudes are counted from the western horizon of this place (i.e. at 90° to the west). It is this second method that he says he uses (Sédillot 1834: 312-13). In the rest of his text he explains how to determine longitudes with the help of tables showing the date and time of the eclipses visible at the Dome of Arîn. The problem is that, as Biot already pointed out in 1841 in his analysis of Sédillot's book, these assertions are not credible. Indeed: on the one hand such tables have never been found; and on the other hand al-Marrākushī gives no means of precisely locating the Dome of Arîn. Thus, in his table, he does not mention any locality whose longitude is close enough to 90° to help this localization.

In fact and in a general way, in all the Arab sources, the geographical position of the meridian of this Dome of Arîn is always extremely vague. I have, for example, found only two mentions in the 12000 locations collected by K&K (including the one in BIR, already indicated, which is an aberration). The determination of this position has given rise to bitter debates, especially in the 19th century, but, as Biot (1841) thought, it is most probably a mythical place, without any real existence, which al-Marrākushī uses here for the rigour of his demonstration.

Which reference meridian ?

Sources after al-Khwārizmī do not appear to have used the Jerusalem meridian as the meridian of reference. In fact, the longitudes of this locality (yellow squares in Fig. 5) show a notable variability, both in Group A and Group C. Conversely, Baghdad (green circle in Fig. 5), capital of the empire, is characterized by a significantly lower variability. But another major locality in the region also shows a low dispersion, namely Damascus (red diamond in Fig. 5). Moreover, the most frequent longitudes of these two localities are expressed by round figures (Damascus A: 60°/ C: 70°, Baghdad A: 70°/ C:

[12] See footnote 5.
[13] KUS, MUF, SML, DST (see K&K for details).

80°). From the 10th century onwards, the meridians of Baghdad and Damascus could therefore have been used indifferently as reference meridians without it being possible to be more precise[14].

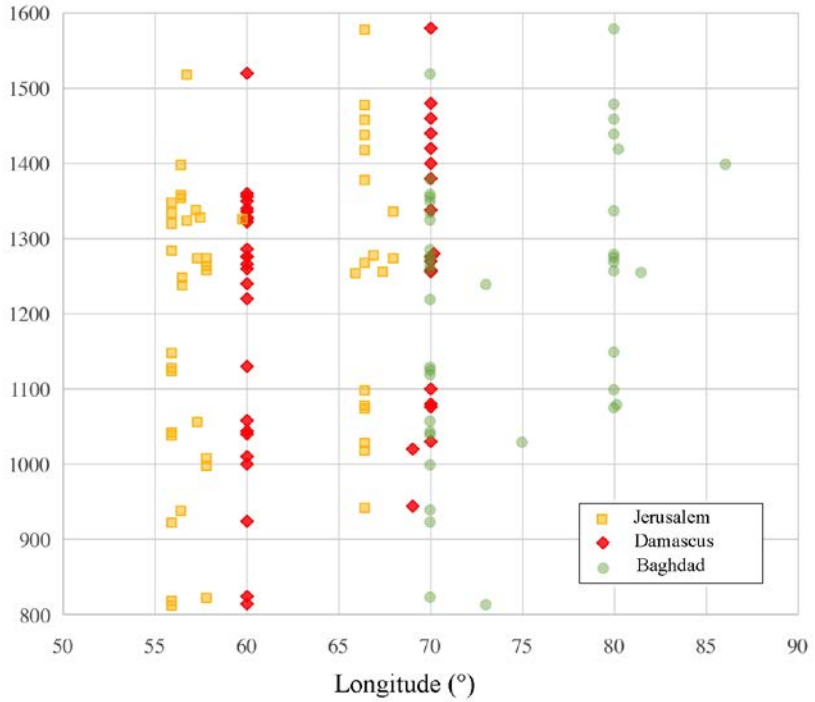

Figure 5: Longitude of three major cities discussed in the text, as a function of the date of the sources. Each city is characterized by a clearly bimodal distribution of its longitude (corresponding to groups A and C). Damascus (in red) and Baghdad (in green) shows a very low dispersion in each of these groups; they can both correspond to the reference (fixed) meridian used from the 10th century onwards.

Review

Contrary to what Arab geographers themselves claim, it is clear that no source listed by K&K has ever used the Atlantic coast as a zero meridian. Moreover, the use of the Canary Islands meridian only refers to Ptolemy and al-Battānī. All the other sources, including al-Khwārizmī, refer to a meridian of mobile water, and totally inaccessible (Comes 1994). In practice the authors therefore had to choose, on land, a fixed meridian whose longitude was historically fixed, to set their geographical tables. This was first the Jerusalem meridian of Ptolemy, probably chosen by al-Khwārizmī (KHU), and then the Damascus meridian and/or the Baghdad meridian as conceived by al-Khwārizmī (all other sources). It seems that the Dome of Arîn, a concept that seems only theoretical, never had a practical role in the determination of longitudes. For historical reasons, the geographical tables present sets of longitude offset by 10°, which allows them to be classified into two groups. In practice, these groups correspond perfectly to groups A & C of Kennedy & Regier (1985), although the defini-

[14]The majority of sources give a round number for both Damascus and Baghdad, which means that it is the heart of the Empire that is considered as a reference, but a few others clearly privilege one of the two cities. For example: KAS, 15th century, seems to retain Damascus (Damascus 70°; Baghdad 80.25°), while al-Marrākushī (MAR) seems to be a proponent of the opposite option (Damascus 70.2°, Baghdad 80°), although he claims, as we have seen, to refer to the Dome of Arîn.

tions proposed by these authors are erroneous. I therefore propose to keep this nomenclature and simply modify the definitions.

Evolution of mathematical geography from the 9th to the 16th century.

Since the measurement of the latitude of a locality has *theoretically* never posed a problem for Muslim geographers, it is indeed, with few exceptions, by studying the longitudes that it is possible to characterize an improvement in the accuracy of geographic data. Furthermore, it has been established above that, despite the fact that the zero meridian has fluctuated enormously over the course of history and that, moreover, there are two systems for quantifying longitude (A and C), *in practice*, Muslim geographers have referred to a Middle Eastern Meridian. It is therefore possible to compare heterogeneous sources by studying, not longitudes, but longitudinal distances from Damascus (which I arbitrarily chose here) or Baghdad.

We have seen that, apart from al-Battānī, all the sources studied have returned the meridian zero to the sea, west of the Canary Islands. This means that they admit a reduction in the longitudinal distance between the Maghreb and the Middle East. Al-Khwārizmī (KHU) was the first to achieve this reduction, which allowed it to correct half of Ptolemy's geodetic error. What about more recent sources ?

Figure 6 shows the evolution of the longitudinal distance between the western end of the Mediterranean Sea [15] (Tanger) and Damascus, these values are to be compared with the Ptolemy value (PTO) and the present value (MOD). It can be seen that very shortly after KHU and BAT, some sources come remarkably close to the correct value. These are, for the oldest, the tables of Ibn Yūnus (YUN- late 10th century), and al-Zayyāt (ZAY – middle 11th century). No link or influence between these two sources can be established; they are independent (K&K, Regier 1987). Ibn Yūnus is a famous astronomer who lived in Egypt (King 2008); his method of determining longitude was studied by Rufus (1931), it is the astronomical method using the Moon eclipses. Al-Zayyāt (c. 951?-c. 1062) is an Andalusian geographer whose main work *Ḏikr al-Aqālīm*,was published and studied by Castelló (1989). In this text, al-Zayyāt does not explain the longitude determination technique he used while he explains in detail the calculation of the latitude (Castelló 1989, pp. 44-47).

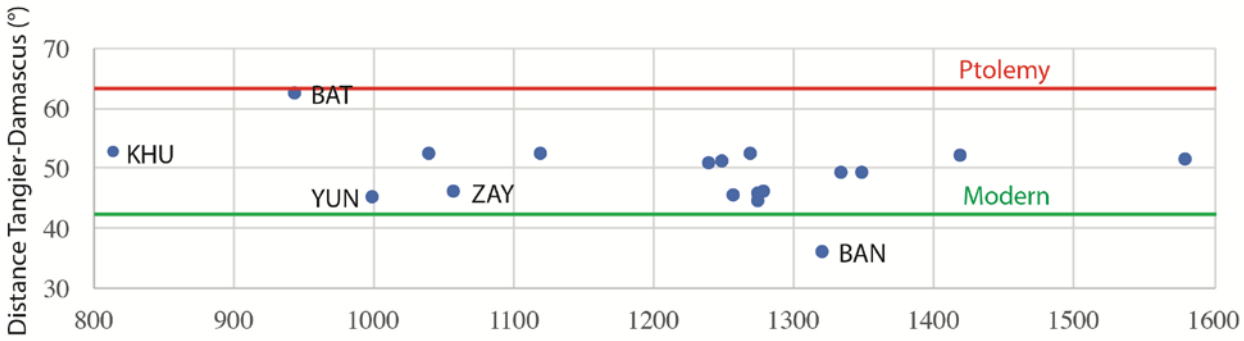

Figure 6 : Longitudinal distance from Tangier to Damascus according to the sources studied and comparison with the Ptolemy value and the current (modern) value. One notes that in the 11th century, with YUN and ZAY a clear progress is noticeable, but as of the 14th century, one finds only values of the order of that of KHU

[15] The longitude of this western limit is calculated here, for each source, as the average of the longitudes of the following localities : Cadiz, Seville, Tangier, Fez and Meknes.

There is also the existence of a source that exaggeratedly reduces the distance between Tangier and Damascus. It is the table of Ibn al-Bannā' (1256-1321) (BAN). This scholar is best known for his work in mathematics, he nevertheless wrote some astronomical works and in his *Minhāj al-ṭālib li-ta'dīl al-kawākib* (Vernet 1952 : 75) he provides us with a table of coordinates for 32 localities (BAN in K&K), but he does not comment on it and gives us no indication of how it was established.
Finally, we note, once again, the lack of influence of al-Battānī with regard to the length of the Mediterranean and, conversely, the very lasting influence of al-Khwārizmī with his moderate reduction. The few authors who have proposed a greater reduction will soon be forgotten. This long-lasting influence is also noticeable through detailed analyses. I will take here the example of the northern coast of the Maghreb, which this time will concern a correction of latitude, at one of the rare places where Ptolemy had made an important error on this coordinate.
Ptolemy had considered this coastline to be rectilinear (Fig 7), with a latitude that gradually decreases towards the East, thus neglecting the deep indentations of the Gulfs of Gabes and Sirte. These systematic errors of latitude over several hundreds of kilometers have long appealed to commentators[16] since, as has already been pointed out here, the measurement of latitude was perfectly mastered at the time of Ptolemy.

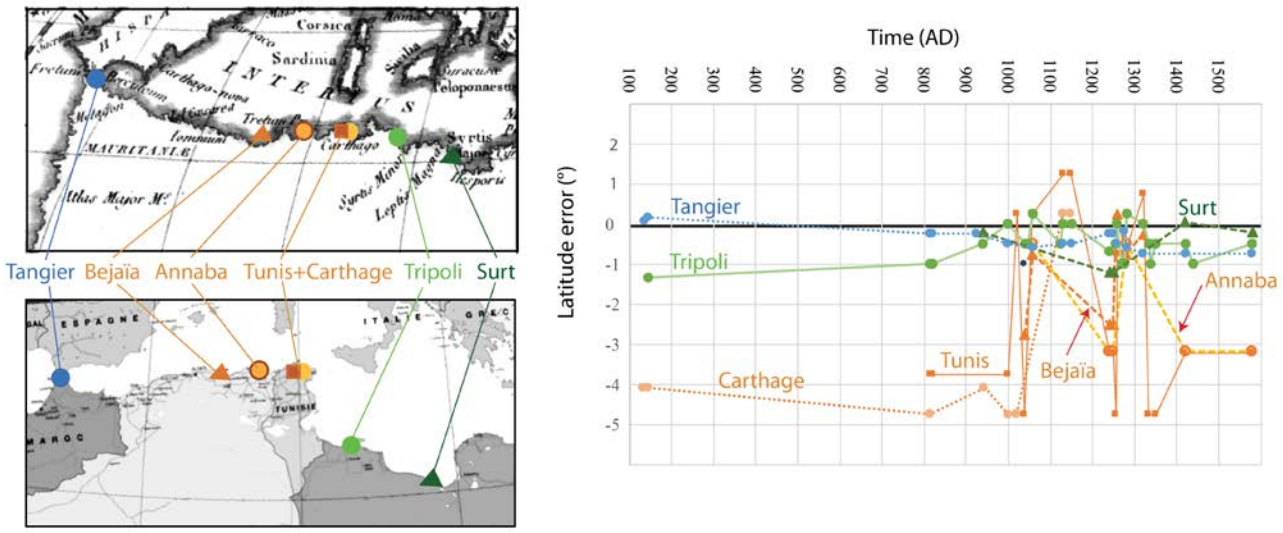

Figure 7 :Evolution of latitude error of seven localities on the southern coast of the western Mediterranean, located on a Ptolemaic map and on a modern map (Sirte = Surt). The erroneous layout of Ptolemy persists until the end of the 16th century; only a few sources from the 11th to the 14th century indicate latitudes compatible with a correct layout. NB: the horizontal scale of the maps is not respected.

But what interests us here is how this error was handled by Arab sources. For this we will compare the evolution of the latitudes of four localities located in the North-West of the Gulf of Gabès (in orange) and two in the South-East (in green). We note that the oldest sources follow Ptolemy globally. This is notably the case of KHU and BAT. Towards the middle of the 11th century, the correction is

[16] In Europe, the first discussions on this subject seem to date back to the beginning of the 16th century (*in* Saada 2016); we will limit ourselves here to a few more recent works: De Rouire (1884), Vycichl (1969), Laporte (2003).

globally made by Al-Zayyāt (ZAY)[17] ; It should be noted that this is one of the first two sources to have introduced a quasi-correct size of the Mediterranean (Fig. 6). Subsequently, and similarly to the correction on the longitude of the Mediterranean, the latitude correction will not be unanimously accepted and, from the beginning of the 14th century, it seems to be totally forgotten because the later sources are in the KHU tradition.

Conclusions

The main conclusion of this work concerns the reference meridian of Arab geographers. Contrary to what they themselves claim, none of them refer to the Ptolemy meridian (except al-Battānī, which is the only one that does not try to correct the "geodesic" error of the latter), nor to the meridian of the Atlantic coast, nor even to the meridian of the Dome of Arîn. In almost all the sources studied, it is a meridian located in the sea west of the Canary Islands that serves as the zero meridian. As this meridian was inaccessible, the authors of geographical tables used a reference meridian that remained fixed from the 10th century onwards: the Damascus meridian or Baghdad meridian. For historical reasons, the sources studied are divided along the longitude of these meridians: 60°/70° (sources A) or 70°/80° (sources C).

The confusion which is pointed out by the previous authors, seems to me therefore very exaggerated. The geographical systems are much less numerous than supposed, I recognize only two of them. Moreover, it is easy to compare sources from these two systems, either by compensating the difference of 10° (Mercier *in press*), or by comparing the longitudinal distances counted from one of the two possible reference meridians.

The result of this work thus constitutes a new framework within which it is possible to make full use of K&K's compilation work, which is very largely under-used, and to renew our approach to the evolution of Arab-Muslim Mathematical Geography.

It is already possible to identify some major characteristics of this evolution:

- al-Khwārizmī (9th century), the founder of the Arab-Muslim Mathematical Geography and incidentally of the A system, corrected some of Ptolemy's errors and had an enormous influence on all sources, including the sources of group C from the 13th century onwards. This influence will be noticeable at least until the end of the 16th century. Contrary to what Al-Bīrūnī and many authors, including very recent ones, have claimed, al-Khwārizmī has never used the meridian of the Atlantic coast as the zero meridian.
- from the 11th century onwards, authors such as Ibn Yūnus and al-Zayyāt will further reduce the errors inherited from Ptolemy and come closer to the current conceptions, at least as far as the size of the Mediterranean and the design of the northern coast of Africa are concerned, which have been considered in this paper.

[17] In fact ZAY indicates 36° for Bijjaya, 36.5° for Buna, 33.25° for Tripoli, but very curiously 27° for Tunis which is clearly a copyist error, I thus restored the 37°. In addition, it should be noted that the famous World Map of al-'Umari (14th century) shows a correct layout of the Gulf of Sirte. It is therefore difficult to follow the late Professor Fuat Sezgin who presented this map as directly inspired (copied) from a Caliph al-Ma'mūn and Al-Khwārizmī map. This is a new argument, after the one mentioned by Rapoport (2020 : 34), that the world map of al-'Umari cannot simply be a copy of a ninth-century map.

- but this improvement of the accuracy of geographic coordinates will not have the success it deserves and will remain provisional, so that from the middle of the 14th century it is forgotten, and one almost returns to the conceptions of al-Khwārizmī.

Acknowledgements

I would like to thank very sincerely Professor Julio Samso (University of Barcelona) who provided me with the computer files that allowed me to reconstruct the K&K database. His kindness is matched only by his enormous erudition.
I would also like to thank the anonymous reviewer, whose constructive criticisms and remarks have enabled me to improve this paper.

References

Ali J. (1973). *The Determination of the Coordinates of Positions for the Correction of Distances between Cities; a Translation from the Arabic of Kitāb taḥdīd nihāyāt al-amākin li-taṣḥīḥ masāfāt al-masākin*; The American University of Beirut 278 p.

Biot J.B. (1841). Commentaires sur le traité des instruments astronomiques; *Journal des savants*: 513-520 & 602-610.

Castelló F. (1989). *Ḏikr al-Aqālīm, (Tratado de geografía universal)*; Consejo Superior de Investigaciones Científicas, Barcelona; 358 p.
Comes M. (1994). The 'Meridian of Water' in the Tables of Geographical Coordinates of al-Andalus and North Africa; *Journal for the History of Arabic Science*, 10, p. 41–51. Repr. in Comes, *Coordenadas del Cielo y de la Tierra.* Universitat de Barcelona, Barcelona, 2013: 377-388.

Comes M. (2000). Islamic Geographical Coordinates: al-Andalus' contribution to the correct measurement of the size of the Mediterranean. In *Science in Islamic Civilization. Studies and Sources on the History of Science*. Istanbul: Research Centre for Islamic History, Art and Culture, Istanbul: 123-138. Repr. in Comes, *Coordenadas del Cielo y de la Tierra.* Universitat de Barcelona, Barcelona, 2013: 389-404.

De Rouire A.M.F (1884). *La découverte du bassin hydrographique de la Tunisie centrale et l'emplacement de l'ancien lac Triton (ancienne mer intérieure d'Afrique)*, Challamel aîné, Paris. 230 p.

Evans K. (1998). *The History and Practice of Ancient Astronomy*; Oxford University Press, Oxford, 480 p.

Kalin I., Ayduz S., & Dagli C. (2014). *The Oxford Encyclopedia of Philosophy, Science, and Technology in Islam*: Two-volume Set; Oxford University Press, 1152 p.
Kennedy E.S. (1973). *A commentary upon Bīrūnī's* Kitāb taḥdīd al-amākin; American University of Beirut, Beirut, 280 p.

Kennedy E.S. (1996). Mathematical geography in Rashed R.: *Encyclopedia of history of Arabic Science*, v. 1: 185-20.

Kennedy E.S. & Regier M.H. (1985). Prime meridians in medieval Islamic astronomy, *Vistas in Astronomy*, 28: 29-32

(K&K) = Kennedy E.S. & Kennedy M.H (1987). *Geographical coordinates of localites from Islamic sources;* Institut für Geschichte der Arabisch-Islamischen Wissenschaften an der Johann Wolfgang Goethe-Universität, Frankfurt; 723 p.

King D.A. (1984). Architecture and Astronomy: The Ventilators of Medieval Cairo and Their Secrets; *Journal of the American Oriental Society*, Vol. 104, No. 1, *Studies in Islam and the Ancient Near East Dedicated to Franz Rosenthal* : 97-133

King D.A (2008). Ibn Yūnus ; in Helaine Selin (ed.): *Encyclopaedia of the History of Science, Technology, and Medicine in Non-Western Cultures*. Springer: 1124-25

King D.A. (2014). *World-maps for finding the direction and distance to Mecca*; Brill edt, 638 p.

Laporte J.P. (2003). Ptolémée et la Maurétanie césarienne, *Comptes rendus des séances de l'Académie des Inscriptions et Belles-Lettres*, 147e année, 1: 171-195.

Mercier E. (*in press*). Mathematical geography in the western Islamic world: geographical coordinates of localities in the al-Maghreb and al-Andalus localities (9th-18th centuries). *Suhayl*

Nallino C.A. (1939). Al-Ḥuwārismī e il suo rifacimento della Geografia di Tolomeo, *Raccolta di scritti editi e inediti*, V, Rome: Istituto per l'Oriente : 458–532.

Nallino C.A. (1899-1907). *Al-Battānī sive Albatenii Opus astronomicum. Ad fidem codicis Escurialensis arabice editum*, Pubblicazioni del Reale osservatorio di Brera in Milano, n. XL, pte I-III, Milan, Hoepli Casa Editrice, 1899-1907 (3 vol.).

Pinto K.C. (2016). *Medieval Islamic maps, an exploration*; University of Chicago Press; 406 p.

Rapoport Y. (2020). *Islamic maps*; Bodleian Library edt., 192 p.

Regier M.H. (1987). Kennedy's Geographical Tables of Medieval Islam: An Exploratory Statistical Analysis; *Annals New York Academy of Sciences*; 500(1): 357-372

Reinaud J. T. & Guyard S. (1848-1883). *Géographie d'Aboulféda*, Paris, Imprimerie nationale, 2 vol.

Robles Macías L.A. (2014). The longitude of the Mediterranean throughout history: facts, myths and surprises*; e-Perimetron*, 9, 1: 1-29.

Rufus C. (1931). Ibn Y*ū*nus' method of longitude determination, *Popular Astronomy*, 39: 527-531.

Saada A. (2016). Reconstitution de l'espace tunisien de Ptolémée et de B. Sylvano (1511); *e-Perimetron*, 11(1): 1-12.

Sédillot J.J. (1834). *Traité des instruments astronomiques des Arabes composé au treizième siècle par Aboul Hassan Ali, de Maroc intitulé «collection des commencements et des fins»* publié par Louis Amélie Sédillot, à l'Imprimerie Royale. Paris. 630 p. + pl

Sezgin F. (2005). *Mathematical geography and cartography in Islam and their continuation in the Occident*, v. 1 from German edition issued as v. 10-12 of Sezgin's *Geschichte des arabischen Schrifttums*. Frankfurt am Main: Institute for the History of Arabic-Islamic Science

Tibbetts G.R. (1992). The Beginnings of a Cartographic Tradition; Cartography in the Traditional Islamic and South Asian Societies, *History of Cartography*, The University of Chicago Press; vol. II-1: 90-108.

von Mzik H. (1926). *Das Kitāb ṣūrat al-arḍ* des *Abū Ğa'far Muḥammad Ibn Mūsā al-Ḫuwārizmī*. by *Muḥammad Ibn-Mūsā* al- Ḫwārizmī; Leipzig: Otto Harrassowitz, 44 + 162 p.

Vycichl W. (1969). Un problème de cartographie historique : Claude Ptolémée et la cartographie de la Tunisie actuelle, *Polyphème (Genève)*, 1: 31-33.