Maria A. Brovelli[*], Marco Minghini[**]

Georeferencing old maps: a polynomial-based approach for Como historical cadastres

Keywords: Cadastre; Como; cultural heritage; georeferencing; historical map; polynomial

Summary
Recent developments in digital technologies have opened new and previously unimagined possibilities for the exploitation of cartographic heritage. In particular, georeferencing converts old maps from pure archival documents to real geographic data. This study investigated the issue of georeferencing the historical maps which are currently preserved at the State Archive of Como. These maps, about 15000 at the scale of 1:2000, belong to different cadastral series: the Theresian Cadastre (XVIII century), the Lombardo-Veneto Cadastre (mid-XIX century) with its updates (1898) and the New Lands Cadastre (1905). Georeferenced maps should then be inserted in the Internet GIS system, developed within the Web C.A.R.T.E. project, for an interactive 2D and 3D consultation.
Due to the peculiar nature of the maps, which are divided in multiple adjacent cadastral sheets for each municipality, a preliminary mosaicking of these sheets was performed. Using the digital cartographic information of current municipalities, Ground Control Points and Check Points were collimated on the historical maps. A polynomial transformation was chosen to georeference the maps. An ad hoc-built procedure based on statistical evaluation of GCPs and CPs residuals was implemented, in order to determine the optimal polynomial order to be used. Evaluation of georeferencing results was performed both qualitatively and quantitatively. The methodology is automated and can be proposed as a reference for georeferencing maps of comparable characteristics. Historical maps can thus be continuously navigated into a georeferenced framework and compared with the current cartographic status. This clears the way for the usage of old maps in a wide range of applications, such as territorial planning, urban and landscape changes analysis and archaeological research.

Introduction

Historical cartography is a significant part of cultural heritage and a valuable source of information for a wide range of applications, such as historical and territorial research, architecture, planning, archaeology and demography. The usage of historical maps in such applications gained a new and revolutionary dimension thanks to the latest advancements in the fields of digital technologies and Geographic Information Systems (GIS), which allowed to study ancient cartography no longer as pure archival documentation, but as real geographic information (Balletti 2006: 32).
However, maps need some important pre-elaboration steps before they can be inserted and analysed in a GIS environment. Users have first of all to think that every map was produced in a particular time and place, thus reflecting specific scientific and geographic knowledge, cultural background or political ideology. It is therefore important to correctly contextualize maps in terms of their date, author, instrumentation used, content and goal of the representation (Azzari 2010: 218). Digitization is then needed to transform from analogical to digital, thus enabling its

[*] Professor of Cartography, GIS and Internet GIS; Politecnico di Milano, DIIAR, Polo Territoriale di Como [maria.brovelli@polimi.it]
[**] PhD student; Politecnico di Milano, DIIAR, Polo Territoriale di Como [marco.minghini@mail.polimi.it]

access, visualization and elaboration by means of digital tools. Due to the possible differences related to map dimension, material, place and state of preservation, digitization has to be carefully planned for each specific case. This involves also an accurate choice of the four acquisition parameters suggested by Fleet (2009): resolution, pixel bit depth, color system and file format.
Finally, the fundamental step for integrating historical maps within a GIS is their georeferencing. This can be defined as the geometrical transformation which associates a metrical framework, i.e. a reference and projection system, to the map image derived from digitization. The increase of map information content after georeferencing is considerable, as a quantitative (i.e. metric) component is added to the original qualitative one (Minghini 2010). Georeferencing provides thus a notable opportunity, allowing to directly integrate historical cartography with every kind of current spatially referenced product and perform on them all the desired GIS analysis.
This research addresses the issue of georeferencing with respect to the cartographic collection of the State Archive of Como (Northern Italy), where about 15000 historical cadastral maps dating back to the 18^{th}, 19^{th} and 20^{th} centuries are preserved. Georeferenced maps will then be inserted in the web based GIS system developed within the Web C.A.R.T.E. project (Web Catalogo e Archivio delle Rappresentazioni del Territorio e delle sue Evoluzioni, *Web Catalog and Archive of the Territory and its Evolutions Representations*). The general aim of the project is to valorize this immense cartographic heritage, performing its georeferencing and building an ad hoc architecture for its 2D and 3D web visualization (Brovelli et al. 2012). This will be used as a new map consultation platform at the State Archive of Como.
More in detail, the purpose of this study is to contribute to the theory of georeferencing historical maps by providing a new polynomial-based approach, which is explicitly developed for the maps under consideration, but can in principle be extended to whatever type of old ungeoreferenced image. The findings are expected to assist historians and map users, providing them with a simple, easily-automated procedure for georeferencing old maps and evaluating their accuracy.
The rest of the paper is structured as follows. First, the main characteristics of the historical maps used in the study are described. Then some background about georeferencing algorithms and evaluation techniques is presented. This is followed by a description of the implemented methodology and the discussion of the obtained results. Finally implications and directions for further research are offered.

The historical maps used

Historical maps preserved at the State Archive of Como depict the large geographical area corresponding to the current municipalities of Como and Lecco districts. These maps date back to four past cadastral series: the Theresian Cadastre (18^{th} century), the Lombardo-Veneto Cadastre (mid-19^{th} century) with the updates of 1898, and the New Lands Cadastre of 1905 (available just for some areas). In the frame of the Web C.A.R.T.E. project, in 2010 the State Archive of Como promoted a high-resolution digitization campaign of the preserved cadastral maps. Digitization was performed through ad hoc flatbed scanners, in accordance with the standard choices of resolution, pixel bit depth, color system and file format (ICCU 2006, Fleet 2009). Figure 1 and Figure 2 show examples of the resulting digitized maps.
The Theresian Cadastre started in 1718 in the territories of current Lombardy by the will of King Charles VI of Habsburg. After the first maps, representing in a geometric accurate way only land parcels (and not buildings) were drawn, the surveys were interrupted due to the Polish and Austrian Succession Wars. Topographic work started again in 1749 under the reign of Maria Theresa

(daughter of Charles VI). Old maps were updated, new buildings maps were produced and all the cadastral registers associated to the representations were compiled. The Cadastre was finally activated in 1760. The Theresian Cadastre constituted an authentic watershed between the previous and the following Italian cadastral productions. As a matter of fact, for the first time in history the central government extended its control up to the single land parcels, thanks to a uniform and impartial taxation method. Maps became then a tool for taxes equalization, as they were specifically produced in order to provide a rigorous and objective representation of reality. These new features promoted Theresian Cadastre as the reference model for all the cadastres that were subsequently activated in the other Italian kingdoms before the national unification (1861).
Theresian maps were separately surveyed for each municipality and produced at the scale of 1:2000 as the composition of multiple map sheets (Fig. 1). The number of sheets varied depending on the municipality dimension. The instrument used for surveying was the plane table, which allowed the drawing of the map directly on the field. No geodetic framework was considered and the problem of projecting the Earth's surface on the plane was ignored; this was acceptable in a first approximation, as the areas to be surveyed (the single municipalities) were small. However, the consequence was that map accuracy usually decreased from the central part of the municipalities, where the surveys began, to the edges, where surveying errors increasingly accumulated (Della Torre 1980).

Figure 1. Example of a municipality map sheet belonging to the Theresian Cadastre (Orsenigo municipality, sheet 8).

The genesis of Lombardo-Veneto Cadastre goes back to the French domination of Northern Italy (beginning of 19th century) after the Napoleonic Wars. The need for a common taxation among all the territories of the kingdom, which included also the Veneto in addition to the old "Theresian" Lombardy, led to a new stage of surveying campaign and map drawing. However, cadastral work was still incomplete when the kingdom went back under the Austrian rule after the

Wien Congress (1815), with the name of Lombardo-Veneto. Assuming the Theresian Cadastre as a model, maps and registers were first produced for the Veneto provinces, in which there was no previous cadastre. Then the work was carried out also in the Lombardy territories, where the Lombardo-Veneto Cadastre was activated much later than the Italian unification, e.g. in 1876 in Como province (Signori 2002). The cadastre, which was updated in 1898 with a completely new series of maps, remained in use until the New Lands Cadastre, i.e. the first common cadastre for the entire Italian area, was activated (1905).

The entities subject to taxation were again the single municipalities, whose maps were divided in multiple map sheets (Fig. 2). The composition of sheets and the territorial extent of each sheet remained unchanged in the updates of 1898 and the New Lands Cadastre of 1905. The survey was again performed using the plane table, but the set up of a preliminary network of points for each municipality was required. In this way, measurement approximations were reduced and the planimetric accuracy of maps increased. Map scale was 1:2000 on the model of Theresian Cadastre; however, some attachments at larger scales (1:1000 or 1:500) were added in order to better represent the most densely-built areas. The decimal metric system, introduced by the French, was used for the first time to achieve a unification of the units of measurement.

Figure 2. Example of a municipality map sheet belonging to the Lombardo-Veneto Cadastre (Orsenigo municipality, sheet 7).

Background on georeferencing

The georeferencing issue results in the application of a coordinate transformation algorithm which links the digitized historical map to a current reference and projection system. In order to define this link, Ground Control Points (GCPs) are needed. GCPs are point features identified on the historical map, for which cartographic coordinates (i.e. coordinates expressed into a current reference and projection system) are also available. These coordinates can be directly obtained from a topographic or GPS survey, but typically they are derived from current digital cartography.

Once GCPs have been collected, a georeferencing transformation can be applied. There are many alternative classifications of georeferencing algorithms: two of the most important separate them into global versus local, and exact versus non exact. Global algorithms make use of all the GCPs, in order to obtain a transformation which is applied on the whole image; local algorithms work instead on finite portions of the image, using each time a different set of GCPs. Exact algorithms make the transformed positions of GCPs coincident with those observed; on the contrary, the exact fitting of GCPs locations is not in principle fulfilled by non exact algorithms. An overview of the most commonly used transformations for georeferencing historical maps is provided by Boutoura and Livieratos (2006). Implementation of each of these algorithms can be found in literature. Similarity or Helmert transformation (Bitelli et al. 2009), affine transformation (Baiocchi and Lelo 2005), 2nd and higher order polynomial models (Bitelli and Gatta 2011) and projective transformation (Balletti 2006) are example of global, non exact techniques. The most used local, exact algorithms are instead thin plate spline (Brovelli et al. 2011), finite element transformation (Balletti 2006), warping and morphing (Guerra 2000).

Performing georeferencing requires also to evaluate the quality of the chosen transformation. The simplest way is a visual comparison between the georeferenced historical map and a current map, by means e.g. of a semitransparent overlapping. Quantitative evaluation involves instead a statistical analysis of GCPs residuals (i.e. the distances between observed and estimated GCPs positions). Especially when implementing an exact algorithm (for which GCPs residuals are all zero), it is also suggested to introduce Check Points (CPs), i.e. points with known image and ground coordinates, which are only used to test the model performance. Statistics on GCPs residuals are a measure of the transformation precision. Conversely, statistics on CPs residuals are indicative of transformation accuracy. The computation of residuals can in turn be exploited in order to generate some intuitive, qualitative visualizations of map deformation, such as displacement vectors and distortion grids (Jenny and Hurni 2011).

Implemented methodology

Mosaicking

The georeferencing procedure on digitized cadastral map sheets required some initial preprocessing steps. Since the final goal will be a continuous navigation of maps in a georeferenced framework, at first it was necessary to accurately cut out the image margin strips, which do not contain any geographical information: at the top and on the left for Theresian Cadastre map sheets (Fig. 1), on the left and at the bottom for Lombardo-Veneto map sheets (Fig. 2).

Then, a careful planning of the georeferencing operation pointed out the need for a preliminary mosaicking of all the map sheets belonging to each municipality and to each cadastral series. This need was primarily due to the aim of generating a unique municipality map for each cadastre, whose GIS management would be easier than the one based on the set of individually georeferenced sheets. In addition, the simple juxtaposition of adjacent map sheets did not generally show a perfect matching between corresponding features located at the borders (streets, buildings, etc.). This suggested that a separate georeferencing of single map sheets would have probably led to a product with scarce metric and semantic continuity, which would not have been suited to the end purpose of Web C.A.R.T.E. project. However, the choice of preliminarily mosaicking cadastral map sheets was mainly due a constraint. In fact, as clearly shown by the enlargements in Figure 3, usually there are external map sheets which contain a very small portion of the municipality map. For this reason, and considering also the case in which the landscape has deeply changed over time, often it is not possible to identify on these sheets even the minimum number of GCPs in order to apply a georeferencing algorithm. The most appropriate solution was therefore to mosaic all the cadastral map sheets of each municipality, which have been previously roto-translated and (whenever necessary) scaled, into a single digital image. Figure 3 shows the obtained mosaic for the Lombardo-Veneto cadastral map of Orsenigo municipality, which is clearly derived from 13 original map sheets. For the sake of simplicity, the results of all the following elaborations will be referred to this municipality, whose Theresian Cadastre map is composed of 9 sheets (Fig. 6) because of a previously smaller dimension of the municipality area. The map of 1898 updates is again the composition of 13 sheets; the New Lands Cadastre map is not available.

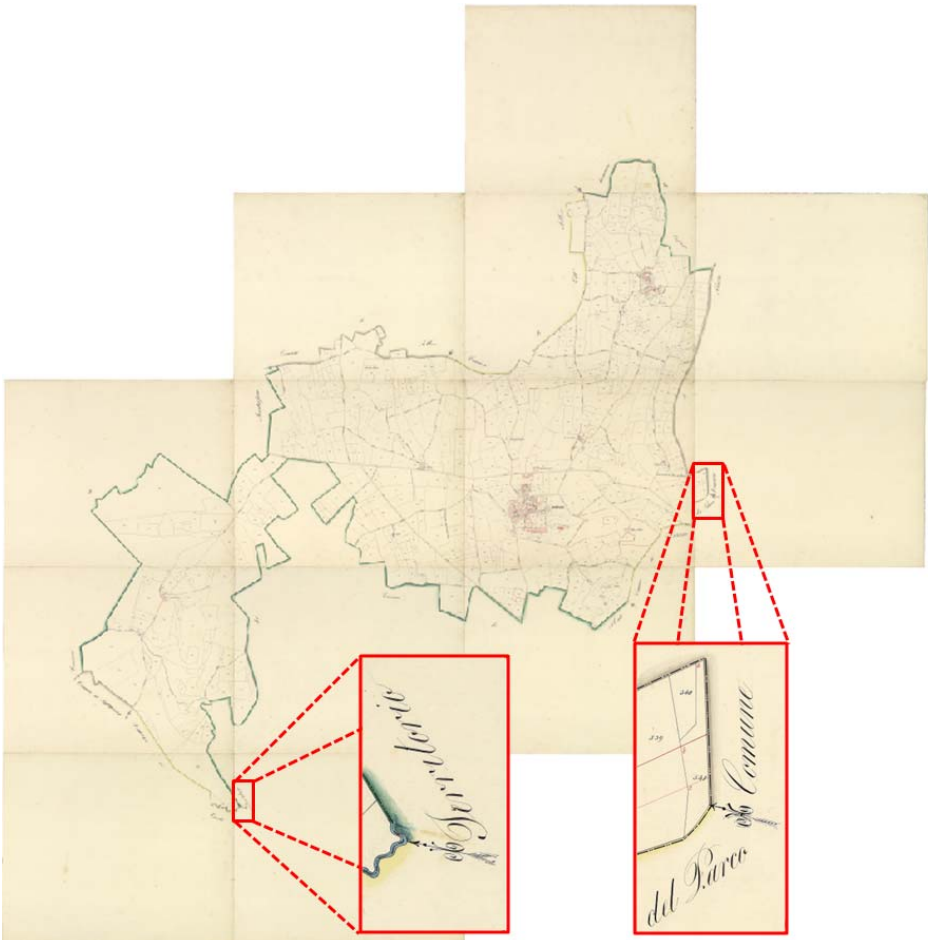

Figure 3. Mosaic of Lombardo-Veneto cadastral map sheets for Orsenigo municipality.
Red-bordered rectangles show enlargements of the small portions of map contained in two external map sheets.

GCPs and CPs collection

GCPs and CPs for georeferencing were derived from the digital cartographic data of current municipalities, which are referenced in UTM WGS84 and are available in scales between 1:1000 and 1:5000. According to the usual cross-validation procedure, approximately the 80% of the points were used as GCPs, while the remaining 20% were used as CPs. All the points were identified as features, which have been preserved almost unchanged over time. Most of them were selected as building edges and municipality boundaries (Fig. 4); some points were instead identified as streets' intersections, especially in the large areas of maps where no buildings existed.

As it can be deduced from Table 1 related to Orsenigo municipality, GCPs and CPs density was usually much greater for the Lombardo-Veneto and the following cadastres, with respect to the Theresian one. The reasons were the less accurate map drawing of Theresian Cadastre and, most of all, the deep territorial modifications which have occurred from that time with respect to the following centuries and to the present. GCPs and CPs were collected as uniformly as possible over the mosaicked map images, according to a random stratified sampling method (Devore and Peck 1993). An example of points' distribution is shown in Figure 6, in which GCPs are represented by green dots and CPs by red dots.

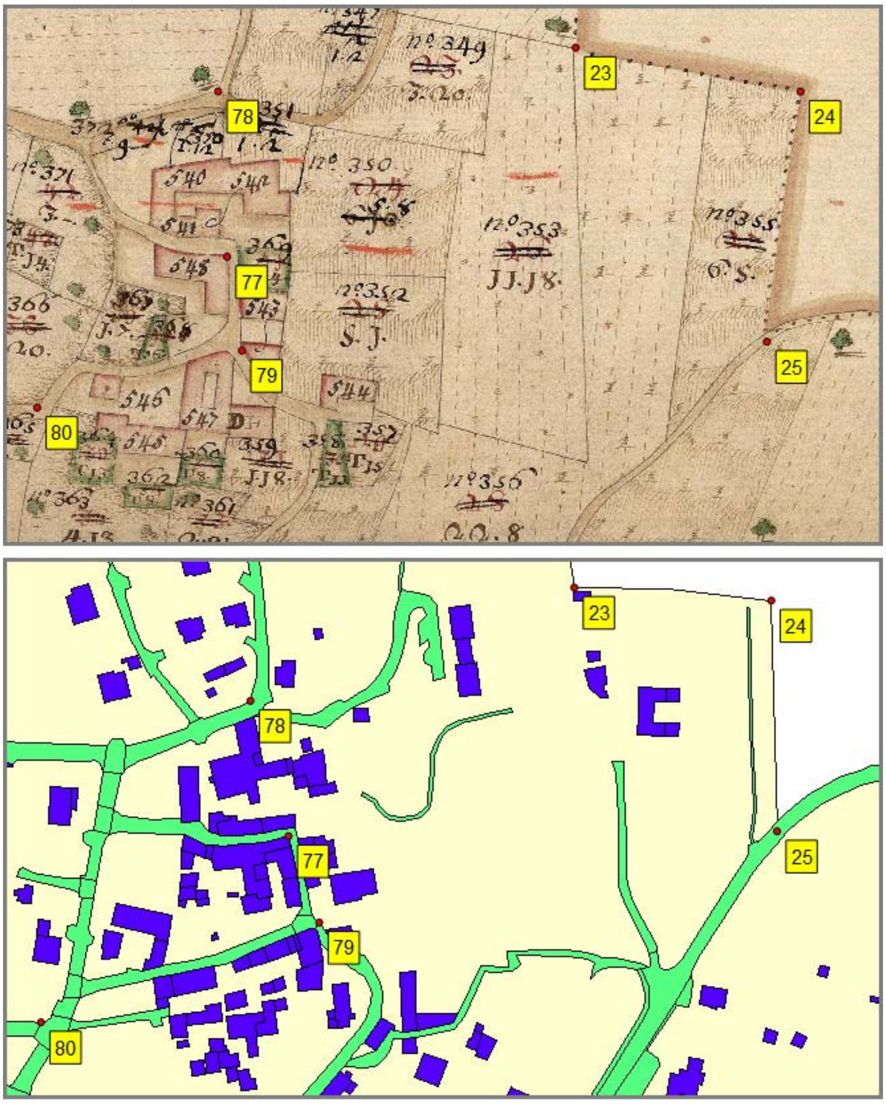

Figure 4. Detail of points collection on Orsenigo municipality Theresian map (top) and Orsenigo digital map (bottom).

	Theresian Cadastre	Lombardo-Veneto Cadastre	1898 updates
N° map sheets	9	13	13
N° GCPs - N° CPs	70 - 16	221 - 56	249 - 62

Table 1. Number of map sheets, GCPs and CPs for the three cadastral series of Orsenigo municipality. Even if the municipality area was smaller in the Theresian map, points density is still much lower than those of the following cadastral series.

Georeferencing technique

As pointed out by Boutoura and Livieratos (2006: 69), the choice of the georeferencing algorithm has to be done according to both the nature of the studied historical map and the final result the analyst wants to achieve. Cadastral maps under consideration were specifically drawn with the best possible metric accuracy, which was needed for the tax equalization purpose they were produced for. This preliminary remark coupled with the consideration that there was no interest in applying an exact georeferencing algorithm, since the perfect fitting of GCPs to their current positions was not needed. On the contrary, the desired georeferenced product was an accurate historical map, in which errors were globally filtered and no local effect was present. For all these reasons the georeferencing choices were reduced to global, non exact algorithms. However, some experimental tests were performed using also thin plate spline, i.e. a local, exact algorithm. Results showed significant local deformations in areas without GCPs (Brovelli et al. 2011), which justified the incorrectness of using models of this kind (Guerra 2000: 340).
The selected georeferencing algorithm was therefore the most used global, non exact one, i.e. the family of polynomial models. For a generic order *m*, the polynomial equations which transform image coordinates (x,y) into ground coordinates (X,Y) are the following:

$$\begin{cases} X = a_{00} + a_{10}x + a_{01}y + a_{20}x^2 + a_{11}xy + a_{02}y^2 + \ldots + a_{m,0}x^m + a_{m-1,1}x^{m-1}y + a_{1,m-1}xy^{m-1} + a_{0,m}y^m \\ Y = b_{00} + b_{10}x + b_{01}y + b_{20}x^2 + b_{11}xy + b_{02}y^2 + \ldots + b_{m,0}x^m + b_{m-1,1}x^{m-1}y + b_{1,m-1}xy^{m-1} + b_{0,m}y^m \end{cases}$$

The total number of unknown parameters a_{ij}, b_{ij} $(i, j = 0,\ldots, m, i + j \leq m)$ is equal to $(m+1)(m+2)$. With a minimum of $(m+1)(m+2)/2+1$ GCPs (which give redundancy), it is possible to apply a least squares adjustment, in order to estimate the parameters. Then the estimated coordinates $(\hat{X}_i, \hat{Y}_i)$ of the i th GCP or CP can be derived, starting from its image coordinates (x_i,y_i). The residual components $RESX_i$, $RESY_i$ along the X and Y directions and the residual module RES_i can be computed as

$$RESX_i = X_i - \hat{X}_i,\ RESY_i = Y_i - \hat{Y}_i,\ RES_i = \sqrt{(RESX_i^2 + RESY_i^2)}\ .$$

All the basic statistics can in turn be computed on these sets of residual values. A global statistics which is typically used for synthesizing georeferencing results is the Root Mean Square Error (RMSE). For X and Y residual components and for residual module, it is computed as

$$RMSEX = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(RESX_i)^2}\ ,\ RMSEY = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(RESY_i)^2}\ ,\ RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(RES_i)^2}\ ,$$

where N is the total number of GCPs or CPs.
The above general polynomial equations include the special cases of affine and similarity transformations, that literature often proposes as stand-alone methods. Affine transformation is noth-

ing but a polynomial model of order $m = 1$, which has 6 unknown parameters. In contrast with all the higher order polynomials, the parameters of a 1st order transformation possess a physical meaning. In fact, an affine transformation is able to model two translations (one in each direction), a global rotation, two scale variations (one in each direction) and a shear angle. If the scale variation is uniform in the two directions and there is no shear, affine transformation simplifies to a similarity transformation, i.e. a conformal 4-parameters transformation. 2nd and higher order polynomial models, which have a gradually increasing number of parameters, can also correct for more complex, non linear distortions.

Each order of the polynomial model could in principle be chosen for georeferencing the cadastral maps. Even though the georeferencing results always depend on the density and distribution of GCPs (Boutoura and Livieratos 2006: 69), a rigorous, well-defined procedure was built to systematically determine the optimal polynomial order for georeferencing. The evaluation criterion had to consider the performances of the generic *m*-order polynomial in terms of both precision (i.e. analysing GCPs residuals) and accuracy (i.e. analysing CPs residuals). Since CPs are not used for model training but just for validation, the accuracy behaviour exclusively depends on those points and not on the polynomial order *m*. In other words, there is no way to know in advance which is the best polynomial order in terms of accuracy. Conversely, the least squares application for different polynomial orders on the same set of GCPs ensures that the sum of the residuals squares gradually decreases, as polynomial order *m* increases. In other words, there is always an increase in precision when moving to higher order polynomials. According to these considerations, the optimal polynomial order for georeferencing each map was chosen as the minimum between the one that produced the lowest RMSE on CPs residuals module and the one that resulted from a statistical Fisher test on GCPs residuals (Minghini 2010). The maximum allowed polynomial order was decided to be $m = 5$.

The Fisher test compared the performances on GCPs for each couple of consecutive-order polynomial models, deducing if the precision increase of the higher order one was significant from a statistical point of view. If so, this polynomial order became the new candidate for georeferencing, and it was in turn compared with the new higher order using the same approach. More in detail, the test had to be performed separately on the *X* and *Y* residuals components of GCPs. For each component, the test evaluated the empirical value F_{emp} of the used statistics, which basically computed the difference between the sum of the residuals squares for the two polynomial orders under consideration. The empirical value F_{emp} was then compared with the theoretical one F_{theo}, which was extracted from the table of *F*-distribution, using a fixed $\alpha = 5\%$ significance level. The decided rule was to move to the higher polynomial order between the two, if this order produced a statistically significant increase in precision on both the *X* and *Y* components. On the contrary, if at least one of the two tests showed a non-significant increase in precision, the lower polynomial order was considered to be the final result of the chain of Fisher tests. As mentioned above, this order (which was decided to be the best in terms of precision) was compared with the one producing the lowest RMSE on CPs residuals module (which was decided to be the best in terms of accuracy). The minimum between the two was finally chosen as the polynomial order to be used for georeferencing.

Results

Least squares adjustments were applied, in order to implement georeferencing polynomial models with orders ranging from 1 to 5 on each municipality mosaicked cadastral map. For all the polynomial orders, mean, standard deviation, minimum, maximum and RMSE were computed on the *X* and *Y* residual components and on the residual module, both for GCPs and CPs. Table 2 and Table 3 show the computed RMSE values (in meters) for the three cadastral maps of Orsenigo municipality, respectively on GCPs and CPs.

	Theresian Cadastre			Lombardo-Veneto Cadastre			1898 updates		
	RMSEX	RMSEY	RMSE	RMSEX	RMSEY	RMSE	RMSEX	RMSEY	RMSE
1st order polynomial	15.41	12.42	19.79	6.60	5.90	8.85	6.98	6.93	9.84
2nd order polynomial	15.32	11.93	19.42	5.06	4.93	7.06	4.99	5.66	7.55
3rd order polynomial	14.17	10.72	17.77	4.99	3.93	6.36	4.83	4.38	6.52
4th order polynomial	12.05	8.85	14.95	4.61	3.83	6.00	4.56	4.20	6.20
5th order polynomial	8.67	5.86	10.47	4.54	3.61	5.80	4.43	3.89	5.89

Table 2. RMSE values [m] of *X* and *Y* residual components and on residual module of GCPs, for the three cadastral series of Orsenigo municipality.

	Theresian Cadastre			Lombardo-Veneto Cadastre			1898 updates		
	RMSEX	RMSEY	RMSE	RMSEX	RMSEY	RMSE	RMSEX	RMSEY	RMSE
1st order polynomial	7.88	6.59	10.26	6.73	6.21	9.16	6.68	6.36	9.22
2nd order polynomial	7.99	6.94	10.58	4.46	5.10	6.78	4.04	5.23	6.61
3rd order polynomial	7.98	8.63	11.76	4.31	3.86	5.78	3.93	3.90	5.54
4th order polynomial	11.57	10.36	15.54	4.06	3.62	5.44	3.71	3.84	5.34
5th order polynomial	13.07	10.89	17.01	3.85	3.27	5.05	3.55	3.83	5.22

Table 3. RMSE values [m] of *X* and *Y* residual components and on residual module of CPs, for the three cadastral series of Orsenigo municipality.

RMSE values allowed appreciating an increase, both in precision and accuracy, of the Lombardo-Veneto cadastral maps and their updates of 1898 with respect to the earlier Theresian Cadastre. This result, which was obtained for all the studied municipalities, quantitatively confirmed the improvement of map surveying techniques from the 18th to the 19th century. The updates of 1898, whose map sheets had been drawn on the basis of their corresponding Lombardo-Veneto ones, showed almost identical precision and accuracy performances. In any case, RMSE values for all the three cadastral series could be considered acceptable, taking into account the nature of maps, the surveying techniques, the deformation of paper supports over time and the previous mosaicking step. The latter inevitably generated larger residual errors than those which would have occurred in the case of a separate georeferencing of each map sheet.

Statistical results on CPs highlighted two different behaviors of the various order polynomial models tested on the maps of Orsenigo municipality. For the Theresian Cadastre map, the polynomial order which produced the highest accuracy was the 1st; for Lombardo-Veneto Cadastre and the updates of 1898, it was the 5th. Looking at the lowest RMSE of CPs residuals module, it was then concluded that these orders were the accuracy-candidates for georeferencing the corresponding maps, as shown in the first row of Table 4. In particular, since 1 was the minimum tested order, 1st order polynomial was obviously decided as the georeferencing algorithm for Theresian Cadastre map.

RMSE values computed on GCPs reflected the principle of a progressive increase in precision with the increase of polynomial order. The Fisher test was therefore performed for each cadastral map and the resulting polynomial orders are listed in the second row of Table 4. The precision-candidate polynomial orders were the 1st for the Theresian Cadastre, the 5th for the Lombardo-Veneto Cadastre and the 4th for the 1898 updates. According to the previously described procedure, the minimum order between the two candidates (deriving from accuracy and precision evaluation, respectively) was finally chosen for the polynomial georeferencing: 1st order for the Theresian map, 5th order for the Lombardo-Veneto map and 4th order for the 1898 map. The results obtained for Orsenigo and for all the other municipalities led to the conclusion that the resulting optimal polynomial order does not generally depend on the cadastral series the map belongs to. It slightly depends on GCPs and CPs distribution and it strongly depends on the peculiar nature of the considered map. However, it was often observed that the optimal polynomial model was the one resulting from the Fisher test. The output of this test could thus be considered as the real constraint for determining the optimal polynomial order, which only rarely went up to the 5th.

	Theresian Cadastre	Lombardo-Veneto Cadastre	1898 updates
accuracy-candidate order	1	5	5
precision-candidate order	1	5	4

Table 4. Best polynomial orders for the three cadastral series of Orsenigo municipality in terms of accuracy (first row) and precision (second row). The final polynomial order for georeferencing each map was the minimum between the two.

Mosaicked cadastral maps were then georeferenced through the implementation of the optimal-order polynomial model. Besides quantitative considerations, georeferencing performance was also qualitatively evaluated by means of a simple comparison with current cartographic products into a GIS environment (Fig. 5). This allowed not only to appreciate the metric accuracy of georeferenced maps, but also to directly perceive the territorial evolution over time.

Figure 5. Semitransparent georeferenced map of Orsenigo municipality 1898 updates superimposed on Google hybrid map.

Another qualitative evaluation of georeferencing quality was obtained from the generation of CPs residual maps (Fig. 6). These representations not only showed the distribution of collimated GCPs and CPs, but allowed also an assessment of CPs residuals by means of vectors having the same direction of the residual themselves and a length proportional to their module. Proportional factor was chosen, in order to ensure the best possible visualization of residual vectors. Residual maps were primarily exploited to verify the absence of systematic behavior of CPs residuals that would have reflected an incorrect choice of georeferencing model. Moreover, these visualizations allowed to appreciate the shift in accuracy between the different areas of each map, often demonstrating an accuracy decrease from the municipality centre towards the edges, which resulted from the survey technique.

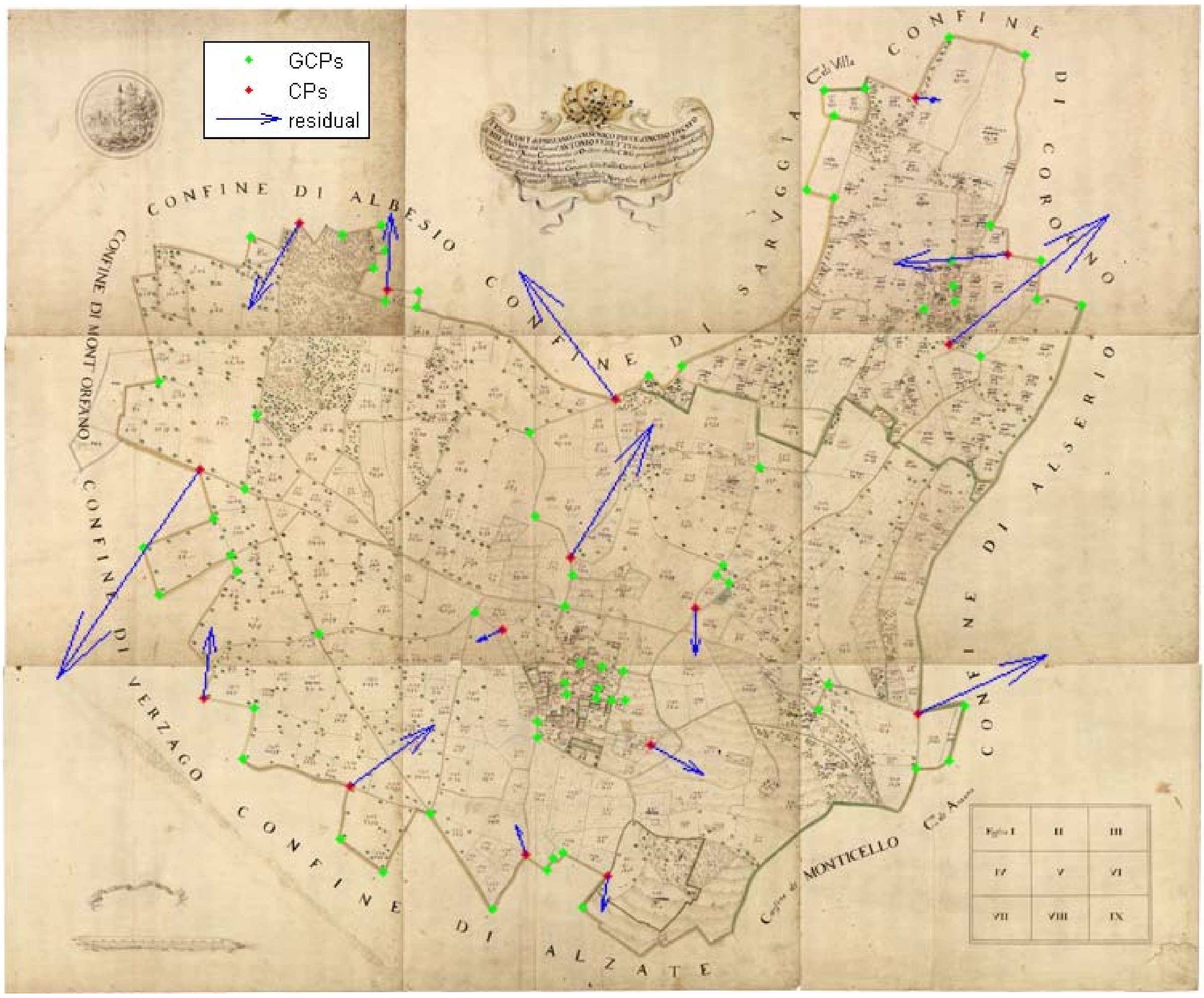

Figure 6. CPs residual map for the Theresian Cadastre map of Orsenigo municipality.

Conclusion

In the framework of the latest advancements in digital and GIS technologies related to cartographic heritage, this study investigated the issue of georeferencing the historical cadastral maps preserved at the State Archive of Como. Georeferencing was the first step for the creation of the Web C.A.R.T.E. (Web Catalogo e Archivio delle Rappresentazioni del Territorio e delle sue Evoluzioni) platform, in which ancient maps are visualized and compared with current cartography in both a 2D and 3D environment.

After a cultural and historical contextualization of the studied maps, a mosaicking of the cadastral map sheets of each municipality was performed in accordance with the final purpose of the project and with the constraints due to the nature of the sheets themselves. A new polynomial-based georeferencing approach was then proposed. This procedure exploits the full potential of the family of polynomial models, which is widely used in literature for georeferencing historical maps (e.g. Baiocchi and Lelo 2005, Bitelli and Gatta 2011). The goal of the methodology is to determine the optimal polynomial order (up to the 5^{th}) to be used for georeferencing each map, on the basis of statistical evaluations of both Ground Control Points and Check Points residuals, i.e. in terms of both precision and accuracy. The results, presented for the case of Orsenigo municipality maps but broadly generalizable, showed that the resulting polynomial order does strictly depend on the peculiarity of the considered map, being thus not a priori definable.

The implemented procedure was automated and can be easily applied by historians and map users, in order to georeference old maps with certainty that the final product will be optimal not only in precision, but also in accuracy (i.e. local deformations are reduced as much as possible). Therefore, although the method was specifically tested on the maps of the State Archive of Como, its flexibility should allow the use for georeferencing maps of comparable characteristics and even any other historical map.

Future work should examine the potential of using current georeferenced cadastral cartography, in order to increase the number of GCPs and CPs (especially in areas without buildings) and in turn improve the performances of the applied polynomial models. The issue is not trivial, as current cadastral cartography was derived from the same 18^{th}, 19^{th} and 20^{th} centuries cadastral maps and it has therefore a significant mismatch, in both geometry and content, with respect to the official current digital cartographic data.

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