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Lafrery's perspective map of Milan (1573): genesis and geometric content

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Summary

This paper considers an analysis of the perspective view of Milan engraved by Antonio Lafrèry and published in 1573. The study focuses on the perspective view in order to define its cartographic content and to establish if it was constructed by a rigorous methodology. In this way we can set the real value of this map: as an artistic or symbolic document or a cartographic document, made by a known projective genesis, which can give us also metrical data.

The study is based on the analysis of the differences between the Lafrery's view and a well-known cartographic product. First of all we georeferenced the maps in a single well-defined reference system with the known plane transformations using the ones with the best results considering the qualitative and quantitative aspects. The georeferencing process has been made using homologous point picked out on a contemporary map and in the perspective view. The urban development of Milan from the XVI century made it impossible for us to discover a large number of points in both maps so we used an intermediate reference map: the one made by the Astronomi di Brera in the XIX century. Having done this we then compared the georerefenced map in order to define the projective genesis of the perspective view. In this task we applied a different kind of representation in order to identify the most immediate and comprehensible way to understand the results.

The digital approach holds an important role because it allows us to process a large amount of data but also to apply all the algorithms of image processing and computer graphics. Moreover it also allows us to obtain different representations such as contour lines map or false color representations or 3d models of the deformation. We tested different software during the georeferencing step to define the best method and algorithm, but we also created some software products to apply different kinds of analysis. In particular we applied the concept of plastic theory to this study and used the strain tensors to locate, in the map, the position of the biggest deformations, both in rotation and dilatation, caused by the georeferencing step.

Introduction

In the study of the city and the territory, historical cartography has always had great relevance to understanding territorial and urban development. Nevertheless, often maps are simply considered as images of a temporary geographic reality, inserted in books as integration and support of different studies, and fundamentally regarded as artistic representations. However, nowadays researchers start to investigate other aspects of cartography in order to understand to what extent it is possible to extract information from a historical map.

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The fundamental query we might ask is, "to what extent is an historical map trustworthy?"

As with contemporary cartography, maps do not have an absolute value, the cartographer sets queries and goals before editing a map. Therefore, his aim is not to depict and record the simple reality. Rather, he wants to provide specific information regarding the function of the map itself. Thus, in his representations, there is a strong and significant component that is symbolic, representative and also conventional.

The reliability of the historical map should be considered depending upon its purpose. In this way, deformations and omissions can be seen as information instead of as errors.

With a pseudo-perspective view, the problem related to representations and survey systems is perceived even more because it extends to the elevations, as well as to the plans.

The main problem is to understand if the perspective construction of the document is rigorous, based on pre-existing maps and plans, or generated as a result of a preliminary survey, and, if so, which instruments and methods have been used.

The idea that the perspective construction was realized starting from plans and elevations is very fascinating for those in the cartography field, because it assumes the existence of a survey of the city or, at least, of a series of reference points positioned on bell towers and registered through triangulation or radiation methods.

The debate on this topic is between two extreme points of view: those who believe it to be pure artwork and those who believe it to be a rigorous perspective plan.

The purpose of this study is to successfully understand the discrepancies of the view compared to a "correct" geometry and therefore to understand which deformations have been introduced by Antonio Lafrery drawing his perspective plan of Milan, published in 1573 in Rome. If the deformation introduced can be normalized by a rule, the view could also be interpreted from a metric, topographic and topologic point of view.

Antonio Lafrèry and the perspective view of Milan

Antonio Lafrèry was an engraver and printer, born in France around 1512, who died in Rome in 1577. He engraved artistic prints and was the first in Rome to deal on a large scale modern geographic maps as well as historical plans and views of cities which were engraved on copper and, for the most part, printed in his workshop.

In the lafreryan view of Milan, one can trace all of the urban history of Milan, up to the Renaissance era. In the center of the urban complex, the ancient system of a Roman *castrum* can be identified, thanks to a series of layouts orthogonal to one another. It is a *castrum* built in the first century BC with a nearly squared shape and 700-800 meters wide. The urban accesses along the city walls corresponded to the ends of the *castrum*'s cross axes. Those accesses then corresponded to the main urban gates created in the medieval and Renaissance walls. Moreover, the medieval walls, with their still intact gates and moat, are easily identifiable, although, clearly, they did not have defensive purposes. Finally, the city is rounded by a new bastioned wall, on the summit of which lies the Sforzesco castle. In 1546, the Spanish decided upon its construction and, two years later, Gian Maria Olgiati begun its construction. Nevertheless, the military equipment of the Sforzesco castle take us a few years back in the city's situation, compared to the time when the perspective view was published. In fact, the angular bastions, which had been built since

1560, in the lafteryan map appear only on the side facing the city. The lafteryan map is extremely useful to understand the original characteristics of the Spanish wall as a build-ing complex, as a type of fortification, and as an object abruptly emerging from the flatness of the agricultural plain, as well as a distinguishing element in several landscape canvases.

In the Renaissance bulwark framing, which encompassed an area of 824 hectares, the urban development took place starting from the beginning of the XIX century. The expansion was almost negligible during the almost two centuries of the Spanish domination. However, it livened up after the second quarter of the XVIII century, during the Austrian domination.

The symbolic contents of the pseudo-perspective view of Milan are easily detectable: Lafrèry uses the off-scale as a symbolic element in this map. The first perceptible characteristic is the size of the Spanish walls. The outer wall, in fact, is highly over-dimensioned and this was a method used to exalt and honor the political power. Moreover, the Renaissance wall is likened to a circular shape, which, assimilating the perspective deformation, becomes ellipsoidal and creates a visual axis going through the Roman Gate, the Broletto and the Castle; axis absolutely non-existent in reality. At the summit of the Spanish wall lies the the Sforzesco Castle, which, in turn, is almost three times the actual size, creating a complete deformation of the urban texture inside the medieval walls, both in the area below the castle and in the area encompassed between the medieval and the Renaissance walls.





Figure 1: The perspective view of Lafrèry (1573). General view and some details emphasizing its symbolism.

Georeferencing

As mentioned above in the introduction, this study aims to define the cartographic content of this perspective view in order to understand if its execution is rigorous, based on preexisting cartography, or resulting from a preliminary survey.

Therefore, this study is directed to evaluate the view's discrepancies compared to a "correct" geometry and to understand which communication tools could be used in order to enable the use of the map by non-experts.

In this regard, technology has a crucial role because it allows, through image processing, the elaboration of a great deal of data and to perform graphic-geometric processing that otherwise would be impossible.

We tested several software programs used for georeferencing in historical cartography, to compare similarities and differences in the processing results in order to select the "best" product.

While georeferencing, the identification of homologous points between the current reference map and the perspective view under analysis is fundamental. As a reference system, we used the numeric cartography of the Municipality of Milan, scale 1:1000 (reference system Rome 40, Gauss-Boaga projection).



Figure 2: The cartographic reference system: the municipal cartography in numerical format (scale 1:1000).

Identifying the points required an intermediate reference map, due to the obvious changes that the city underwent from the XVI century to the present day. We chose a map dated XVIII-XIX century and edited by the Astronomers of Brera, scale 1:1000, divided in 39 sheets and georeferenced for preliminary studies,¹ but geometrically adjusted to allow a better comparison with the map under study. The identification of homologous points is an essential phase for further processing and their distribution, which must be homogeneous within the image to avoid systematic errors in the analytical analyses, may compromise final results.

¹ Monti C. et al. 2009



Figure 3: The intermediate reference map: the map of the Astronomers of Brera (scale 1:1000).



Figure 4: Identification of the homologous points (first phase) between the Municipal and the Astronomers' maps.



Figure 5: Identification of the homologous points (second phase) between the Astronomers' Map and the view of Lafrèry

Thus, it was a matter of identifying buildings and sites that could be recognized with certainty in the views of Lafrery and the Astronomers of Brera and also the current view. The main difficulty of this phase is related to the fact that the city underwent numerous and apparent architectonic and urban changes. Twenty-eight corresponding points have been identified with certainty between the View of the XVI century and Map dated XIX century of the Astronomers of Brera. Most of the identified points lie on the bastions of the Renaissance walls and this highlights the impossibility of georeferencing directly form the Lafrèry's View onto the Numerical Municipal Cartography, since, at the present time, in Milan only a few fragments of the walls in question still exist, making impossible to collimate them as homologous points.

Map georeferencing occurs through the well known procedure² that entails the implementation of geometric planar transformation to achieve a good overlapping between the historical and the current map.

The choice of the transformation depends upon the goals and the cartography type. To georeference/compare the map in its whole or to estimate the distribution of the deformations, a global transformation is suggested, since it implies global translation/ rotation/ rescale. On the other hand, when the analysis focuses on specific topographic-topologic elements, or on specific parts of the map, a local transformation is recommended. The op-

² Balletti C. 2000, Balletti C. et al. 1999.

timal procedure consists of applying a global transformation to eliminate the most significant quantitative differences and, subsequently, to apply a local transformation for a "polish". As global transformation models we used the affine transformation (six parameters) for the map of the Astronomers of Brera, and the projective transformation (eight parameters) for the Lafrery's map.

ERMapper (specifically, to mosaic the georeferenced images of the different sheets composing the maps) and *ArcGis* were the software applications used during the georeferencing procedure. In this environment it is possible to apply a transformation called *adjust*, useful when optimization between global and local accuracy is needed. Such transformation implies an affine transformation and, subsequently, a local "polish" of the control points of the old *map* to better adapt them to those on the *new map*, using a TIN interpolation (Delaunay triangles).



Figure 6: Application of the transformations using the geometry of the map of Lafrèry as reference map, to highlight the deformations in the construction of the perspective. From the use of the projective transformation (left), we gather that there is a different scale that decreases from left to right. This kind of deformation has been probably introduced to confer symmetry in particular to the geometry of the walls. From the application of the finite element transformation (right) onto the Map of the Astronomers of Brera, (the overlapped grid), we notice small rotations within the urban texture. One is particularly important and corresponds to the area that goes from the Dome to the medieval walls. In fact, at a first glance, one can notice that in the Lafrèry's view the Dome has an orientation that is rotated clockwise compared to the real one.

After performing the transformations, the study addressed the analysis of the residues resulting from each transformation to evaluate and interpret, based on their dimension and distribution, the outcomes, as shown in the following schemes. Considering the residuals obtained and their distribution, we decided to carry on the georeferencing process from the transformed Lafrèry's Map accordingly to the affine transformation. Even though the residuals obtained were the highest, we observed that the distribution of the residuals is the most coherent with the conformation of the initial map. In fact, the highest residuals correspond to one of the bastions, precisely the North one, which results clearly rotated compared to the proper orientation of the walls of Milan. The lowest residuals, instead, correspond to an area with the greatest concentration of reference points. This over-parameterization phenomenon is typical of the least squares method.



Figure 7: Analysis of the residual obtained after applying the global geometric transformations.



Figure 8: Geometric correction by using finite element.

Conclusions

As a result of the georeferencing processing, we were able to reconstruct the plan of the city of Milan dated 1560. The resulting map allows us to understand and to evaluate the historical and urban developments of the city in a direct and clear fashion. The errors induced by Lafrèry in the projection model, emphasized by the geometric analysis performed, highlight the reason why the map was created in the first place: to fulfill commemorative and educational purposes, hence, to show the power of the city of Milan through its aspect.

The results obtained do not represent the close of this study. Development opportunities, currently in a concluding phase, concern on the one hand, a further analytic elaboration through tensor analysis to identify the parameters that best represent the introduced deformations. On the other hand, they regard the investigation of the most updated representation models, capable of grasping the urban structure of Milan in the XVI century using a contemporary interface. The goal indeed, is to build a city model based on the map resulting from the geometric transformations that have a current geometrical content, but also a semantic content referred to the cartographic representation of Lafrery. Thus, it is a tridimensional virtual representation of the city in the XVI century, georeferenced and viewable on the web via Google Earth, and therefore accessible by an increasing number of users, in favor of a wider spread of the cartographic culture.

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