Spatial distribution of Ptolemy’s *Geographia* coordinate differences in North Mediterranean eliminating systematic effects

*Keywords:* Ptolemy’s *Geographia*; Ptolemy’s coordinates; Ptolemaic reference system; best fitting; spatial distribution of longitude and latitude differences.

**Summary**
Claudius Ptolemy, in his *Geographia* describes geographic sites (i.e. towns, mountain picks, river mouths, promontories and other) as points with given coordinates of spherical longitude and latitude type. These geographic coordinates are following the known Ptolemaic reference system of parallels and meridians, the origin of which is respectively close to actual Equator and close to the Canary Islands many degrees west of the today’s origin at Greenwich. It is also known that though latitudes are rather well defined, considering the level of measuring accuracy at Ptolemy’s times, the longitudes suffer severe shortcomings which are due to the difficulties of measurement time, which corresponds to the longitude.

In this paper, part of a broader research carried out the last years by the Cartography Group of our Faculty, we focus our interest on Ptolemy’s coordinates given in *Geography* for North Mediterranean, putting together all the partial results came from previous research for every single region in this area. Storing digitally the coordinates for the area of interest, and snooping the data, a laborious process requiring the cross-checking with the relevant coordinates given in a number of Ptolemy’s *Geographia* editions, the finally accepted list is formed which is compared with their today’s values. The core of the study concerns a two-dimensional spatial analysis of the field of differences, testing various transformation functions in order to determine and eliminate the systematic error pattern inherent in Ptolemy’s coordinates. The result, using new “reductive” methods in the comparison analysis (e.g. the concepts of the unit sphere, of the common projective support) with all affined illustrations of the associated test, shows the pattern of coordinate differences free of systematic effects up to the 2nd order. Finally, the spatial deformations of isotropic and anisotropic character for every region of North Mediterranean, is once more, tested and visualized.

**Introduction**

The interest in the geometric properties of historic maps has never been exhaustively and continuously treated by analytical means, especially in the modern era of cartography. The analytical treatment of the geometric background of early maps is an issue that today attracts the attention it deserves, as a result of the challenging perspectives opened by new digital technologies. These new technologies offer generously adequate processing tools that allow diving into the world of the geometric origin and properties of historic cartographic representations and maps.
Previous research showed the order of magnitude of the longitude and latitude differences of Ptolemy’s values from the today’s counterparts both in broader and local scale (Livieratos, 2006), diving into a systematic geodetic approach on the issue for the area of actual Greece and Asia Minor (Tsorlini, Livieratos, 2006) and Spain (Tsorlini, 2008). The core of this study concerns a two-dimensional spatial analysis of the field of differences, testing various transformation functions in order to determine and eliminate the systematic error pattern, inherent in Ptolemy’s coordinates for the Italian peninsula, the islands around it, Sicily, Sardinia and Corsica islands and the Adriatic coast. The result, using ‘reductive’ methods in the comparison analysis (e.g. the concepts of the unit sphere, of the common projective support) with all affined illustrations of the associated test, shows the pattern of coordinate differences free of systematic effects up to the second order. Moreover, in end, a comparison of the patterns of longitude and latitude differences between North Mediterranean and each region in it separately, is attempted giving the appropriate conclusions.

**Ptolemaic reference system and coordinates**

In *Geographia*, Ptolemy gives a list of geographic coordinates of spherical longitude and latitude of almost ten thousand of point locations, on the earth surface, as known at his times. These points are referred to geographic sites (i.e. towns, mountain picks, river mouths, promontories and other) and their geographic coordinates are following the

![Diagram](image)

**Figure 1.** The origin of parallels and meridians in Ptolemy’s *Geographia.*
known Ptolemaic reference system of parallels and meridians, the origin of which is respectively close to actual Equator and close to the Canary Islands almost 18 degrees west of the today’s origin at Greenwich (Figure 1).

**Coordinates in Ptolemy’s *Geographia***

The world of Ptolemy is classified in Regions, since each chapter is referred to one of them, giving by this way the Atlas concept. The smaller the table is the more important and detailed the region appears to be in Ptolemy’s *Geographia*, as it is obvious from the next Figure (Figure 2).

![Figure 2. The ‘Tabulae’ in Ptolemy’s *Geographia*.](image_url1)

![Figure 3. Table V in Ptolemy’s *Geographia* (DeTurre, 1490) with Illyria and Dalmatia depicted on it.](image_url2)
In this paper, at first, which is a part of a broader research carried out the last years by the Cartography Group in the Faculty of Surveying Engineering at Thessaloniki, we focus our interest, at first, on Ptolemy’s coordinates given in *Geographia* for Adriatic coasts listed in Book II, Chapters XV and Italian peninsula and the islands around it, Corsica, Sardinia and Sicily, listed in Book III, Chapters I to IV. These regions are depicted in Tables V, VI and VII of Europe, as it is obvious in Figures 3, 4 and 5. It is five different regions in
Ptolemy’s Geographia, named as Illyria or Liburnia and Dalmatia, Italia, Corsica Insula, Sardinia Insula and Sicilia Insula.
In this case, we are talking about almost 710 pairs of coordinates. The editions, we use for this particular study are the following four:

1. the *Vatopedion Codex* (13th-14th century),
2. the *Marciana Codex* (15th century),
3. the *Donnus Nicolaus Germanus* mid-15th century manuscript of Ptolemy’s *Geographia* as given in Codex Ebnerianus (Stevenson 1991: 92) and

**Processing the Ptolemy’s Coordinates**

According to the procedure we follow, which is depicted in the Figure 6, we first collect the coordinates of the area of interest from the different editions of Ptolemy’s *Geographia* we use, we transcribe them from Byzantine writing, if it’s necessary, and then, we store them digitally in a database (Table 1), having by this way the digital cataloguing of geographic coordinates.

The coordinates from the four sources, are independently and mutually checked and evaluated through this database, in order to detect discrepancies in the point placement, gross errors, double values an edition may have for the same toponym (Table 2), or lack of values and toponyms in some editions (Table 3).
The next step is the projection of the toponyms onto a map with a relevant graticule of parallels and meridians, all of them plotted in the same projection, e.g. the elementary geographic projection \( y = R\phi, x = R\lambda \), assuming a unit radius reference sphere \( (R = 1) \) for the earth’s model. In this process, maps are plotted from the coordinates and by this way, the locations of points are visualized, making easier the auto- and cross-checking of the values, the detection of the differences, the gross errors, the double values and other
displacements they may occur. There is a figure below (Table 4) that shows, as an example, a part of the gross errors in all editions, as they were detected in the database.

Table 3. Lack of toponyms in the editions.

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<th>Toponym</th>
<th>Ben Ben</th>
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<th>Nascina Coda</th>
<th>Marc Marc</th>
<th>Map Map</th>
<th>Valpaxton Coda</th>
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Table 4. The gross errors in the editions used.

The projection of toponyms to a map for Germanus edition, as an example, before their correction from gross errors, is shown in the image below (Figure 7). The different colors of the points are referred to the different regions described in Geographia.
Having projected the coordinates on the map, we compare and analyze them using geodetic methods, so as to try to conclude to an accepted list without gross errors, double values or records or other apparently erroneous discrepancies in point placement, which is also projected onto a map in the same projection (Figure 8).
The identification of ancient toponyms with their modern counterparts

Another important procedure in this study is the identification of ancient toponyms with their modern counterparts. Having concluded to an ‘accepted’ list of coordinates without gross errors, double values or records or other apparently erroneous discrepancies in point placement, we start the comparison of Ptolemy’s coordinates with their today’s counterparts. In order to perform such a comparison and to identify the coincidence of places in Ptolemy’s era with their today’s counterparts, we had to compare the toponyms of each region of Ptolemy’s Geographia with the toponyms of the corresponding area these toponyms are located nowadays, confirming at the end the coincidence of the with certainty known points, based mainly on old maps and relevant references in historical and archeological sources.

In this study, the whole inquiry of maps, old and modern, and of the other historical data, collected and used for the identification of ancient toponyms with modern, is based on internet. The criteria used for the selection of the maps are mainly their resolution - the bigger resolution the map has, the better and easier it can be read after its fitting to the graticule, plotted for the area of interest - and the existence of geographic graticule on the map, which helps maps’ fitting to the ‘plotted’ graticule. With regard to old maps, it is also important to check three more things before the selection of an old map, the prime meridian, which is better to be in Greenwich, so that the control points used in the best fitting process can be easily found, the toponyms on the map written with their ancient names and the modern coastline. In that way the modern places where the ancient toponyms are located, can be more easily detected since the old map’s coastline will fit exactly to the modern coastline (Tsorlini, 2008).

![Figure 9. The best fitting of modern and old maps to the graticule plotted in the area of actual Italy and the islands around it.](image-url)
Based on these criteria, at first, modern maps are selected and fitted with the best possible way to the ‘plotted’ graticule, using as control points, the common nodes of the graticule. The coastline of the modern map is digitized and then we follow the same procedure with the selected old maps. The procedure followed in this case is shown in the figure 9.

The old maps, selected and fitted to the modern map in order to help to the identification of ancient toponyms with modern are:

a. Italy, Alex Findley, published by T.Tegg, London, 1847 (Figure 10a)
b. Ancient Italy, Francis Paul Becker, 1843, engraved by the Omnigraph, F.P. Becker & Co. Patentees (Figure 10b)
c. Italic pars Septentrionalis - Northern Italy, Atlas Of Ancient And Classical Geography, J. M. Dent And Sons, 1912 (Figure 10c)
d. Italic pars Media - Central Italy, Atlas Of Ancient And Classical Geography, J. M. Dent And Sons, 1912 (Figure 10c)
e. Pannonia, Dacia, Illyricum, Moesia, Macedonia et Thracia, Keith Johnston, 1866 (Figure 10d)

Figure 10. Old maps fitted on the modern coastline (a) Italy, Alex Findley, London, 1830, (b) Ancient Italy, Francis Paul Becker, 1843 (c) Italic pars Septentrionalis and Italic pars Media, Atlas Of Ancient And Classical Geography, J. M. Dent And Sons, 1912, (d) Pannonia, Dacia, Illyricum, Moesia, Macedonia et Thracia, Keith Johnston, 1866

Having compared the toponyms, the ancient with modern, we concluded to have almost 400 identified points, without counting on them the mountains and some physical borders,
Ptolemy included in his Geographia. Most of these points will be used as control points in best fitting Ptolemy’s map to the modern map of this area. In the next map (Figure 11), we can see on a modern map, the places, where most of Ptolemy’s toponyms in the area of interest, are detected according to historical and other sources.

![Figure 11. Ptolemy’s toponyms depicted on a modern map](image)

**Best fitting of Ptolemy’s representation to a modern map**

The points, we have mentioned before, have great importance to the continuity of this work because a set of them, properly distributed to the overall map space, is selected and brought into one to one correspondence with the actual coordinates of the same set of points in the modern map, after choosing a transformation system, in this case a 2\textsuperscript{nd} order polynomial transformation, involving a projection and an earth’s model. The result of the best fitting of Ptolemy’s coordinates to the modern counterparts is shown in Figure 12. The Ptolemy representation is georeferenced to actual geographic coordinates using almost 300 control points properly distributed in the area of interest. Ptolemy’s graticule, extended from 26° to 47° in longitude and from 35° to 48° in latitude, contrary to the geographic graticule of the modern map, which extended from 6° to 20° in longitude and from 35° to 48° in latitude. In the resulting map (Figure 12), Ptolemy’s map of coordinates is transformed into the actual coordinates and the deformation appeared in Ptolemy’s graticule is obvious.
The spatial distribution of differences in longitude and latitude

Using the best fitting of Ptolemy’s representation to the modern map, we study also, the spatial distribution of the differences in longitude and latitude induced after the comparison of Ptolemy’s coordinates with their actual values. In the next two figures, which depict the distribution of the differences in both cases (Figure 13 and 14), it is obvious that the distribution is not the same.

As we can see below, the longitude differences vary from 18° on the west, at the Italian borders with South France, to 25.5° at the southeast side, at the Adriatic coast, whereas the latitude differences are of much smaller magnitude than those of longitude and vary from -2.5° at southwest, in Sardinia island to almost 0.5° at southeast. These differences can be easily explained by the fact that though latitudes are rather well defined, considering the level of measuring accuracy at Ptolemy’s times, the longitudes suffered severe shortcomings which are due to the difficulties in measuring the time, which corresponds directly to longitude. Moreover, the longitude values given by Ptolemy are strongly dependent upon the distance from the Canaries eastwards.
Figure 13. The isolines of longitude differences, in degrees, between Ptolemy’s values and their actual counterparts

Figure 14. The isolines of latitude differences, in degrees, between Ptolemy’s values and their actual counterparts
The best fitting process of Ptolemy’s map to the modern and the spatial distribution of coordinate differences in North Mediterranean

Attempting to came to a conclusion about magnitude of the longitude and latitude differences of Ptolemy’s values from the today’s counterparts both in broader and local scale, and having already the results from the spatial distribution of coordinate differences in every region of the North Mediterranean separately, we follow the same procedure to study uniformly the spatial distribution of coordinate differences in North Mediterranean.

According to this procedure, depicted in the diagram above (Figure 15), we use the accepted list of Ptolemy’s toponyms from each region of the area of interest separately and we project them onto a map in the same projection (geographic projection, assuming a unit radius reference sphere as the earth’s model) (Figure 16).

Figure 15. The Ptolemy’s *Geographia* project referring to the spatial distribution of differences in longitude and latitude in North Mediterranean.

Figure 16. Coordinate plotting in geographic projection according to the accepted lists of Ptolemy’s coordinates for each region of the area of interest.
In point of the identification of the coincidence of places in Ptolemy’s era with their today’s counterparts, we use the identified point locations came from the comparison of Ptolemy’s coordinates with their today’s counterparts performed from each region separately in previous studies (Figure 17). These identified toponyms, almost 1200 in the overall area, are all projected to the modern map in the same projection and the visualization from this process is shown in the figure below (Figure 18).

A set of these points, almost 300, properly distributed to the overall map space, is selected and brought into one to one correspondence with the actual coordinates of the same set of points in the modern map, after choosing the same transformation system, the 2\textsuperscript{nd} order polynomial transformation, involving a projection and an earth’s model. The result of the best fitting of Ptolemy’s coordinates to the modern counterparts and the deformation appeared in Ptolemy’s graticule are shown in Figure 18.
Finally, using the best fitting of Ptolemy’s representation to the modern map, we study the spatial distribution of the differences in longitude and latitude in North Mediterranean, induced after the comparison of Ptolemy’s coordinates with their actual values. In the next two figures, which depict the distribution of the differences in longitude and latitude (Figure 19 and 20), it is obvious, as it is expected, that the distribution is not the same.
In an attempt to compare the isolines of the coordinate differences between Ptolemy’s values and their actual counterparts, we notice that the range of the values is the same both in local and broader scale, though the pattern of the isolines differs in global scale from that showed for each region separately, as it is obvious in the figures below (Figure 21-22). That is explained from the interpolation of the points used in each case, which is global in the case of North Mediterranean and local in the case of each region, on its own.

Figure 21. The isolines of longitude differences, in degrees, between Ptolemy’s values and their actual counterparts in North Mediterranean and in Spain, Italy and Greece.

Figure 21. The isolines of latitude differences, in degrees, between Ptolemy’s values and their actual counterparts in North Mediterranean and in Spain, Italy and Greece.
Concluding remarks

The advances of digital computational and visualization technologies are massively available today allowing new approaches and techniques in studying this extraordinary document of our cartographic heritage as it is the Ptolemy’s Geographia. The transformation of early maps into digital form and their comparison with modern maps using new processing methods and technologies is of great importance for the study of the geometric properties of early cartographic documents. Best fitting techniques are appropriate in order to compare early cartographic representations with their modern counterparts.

This study particularly, as well as previous research, both in broader and local scale, showed the order of magnitude of the longitude and latitude differences of Ptolemy’s values from the today’s counterparts. The result of the two-dimensional spatial analysis of the field of differences in Ptolemy’s coordinates shows the pattern of coordinate differences free of systematic effects up to the 2nd order. The comparison of this spatial distribution between areas of broader and those of local scale showed that the range of values remain the same, though the pattern sometimes differs. This work is still in progress and it is extended by testing also and some higher order effects in order to get a better understanding of the whole process.

References


Tsorlini A., E. Manoledakis, C. Boutoura, E. Livieratos 2007: Thessaloniki Project on
Ptolemy’s *Geographia*, 22nd International Conference on the *History of Cartography*, Berne, 8-13 July.

Tsorlini A., E. Livieratos 2007. Digital Approaches in eliminating the higher order systematic effects in Ptolemy’s ‘Geographia’ longitude and latitude differences, XXIII International Cartographic Conference, Moscow, 4-10 August


**Editions of Ptolemy’s *Geographia***

Marciana Codex, 15th century (Codex CCCLXXXVIII) in facsimilae, Athens: Militos Publ.


Vatopedion Codex, 13th century, in facsimilae, Athens: Militos Publ.


De Turre, Tabulae V, VI, VII of Europe, Ptolemy’s Geographia, Rome, 1490

**Ancient maps from internet**


Ancient maps can also be found in the following urls.

[www.lib.utexas.edu/](http://www.lib.utexas.edu/)
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