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Ptolemy’s Geographia in digits

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Summary

Claudius Ptolemy, in his Geographia, gives a list of geographic coordinates of spherical longitude and latitude of almost ten thousand point locations on the earth surface, as they were known at his times. The list is organized in Tabulae which correspond to specific regions of the three known continents at that time, Africa, Asia and Europe. Research on Ptolemy’s Geographia has started at the University of Thessaloniki, Greece, in the eighties, focused mainly, but not exclusively, on data related to territories which are now under the sovereignty of the modern Greek state. This research is concentrated on the study and analysis of Toponymy, in one hand, but more specifically, on the intrinsic geometric structure given in Geographia, parameterized by the arrays of spherical geographic coordinates of longitude and latitude assigned to points on the earth surface apparently known at Ptolemy’s times. The research concerns: a) the geometric properties related to the complete sets of coordinates of point locations (towns and points on natural features, as rivers, mountain crests, promontories etc.) which are identified, or not, today, and b) the use of only the identified location of points in order to approximate non-identified locations (in Ptolemy’s sense) on modern geographic layouts, assisted by relevant archaeological analyses and evidences. The “Thessaloniki Project” on Ptolemy’s Geographia is highly supported by the application of a manifold of geodetic methodologies and techniques, which are coupled with modern digital tools allowing powerful and prompt computational interactivity and visualization of the results in graphic environment, important factors in the analytical processing. The research is also supported and enhanced by the extensive comparative use of old maps created on the basis of Ptolemy’s Geographia, a process which permits the pure cartographic extension and enrichment of the project offering in addition insights from the relevant study of Geographia-born maps.

Introduction

Geographia is the fundamental Greek work of the Roman era, written by Claudius Ptolemy which described for the first time, textually and numerically, the geography of the known World in the 2nd c. A.D. In this work Ptolemy gives the know-how on map construction and also a list of geographic coordinates of spherical longitude and latitude of almost ten thousand point locations, on the earth surface, known at his times. In this paper, which is based on the research carried out the last years by members of the Cartography Group at the Faculty of Surveying Engineering, University of Thessaloniki,
the interest is focused on Ptolemy’s coordinates given in *Geographia* for the area corresponding today to the territory of modern Greece. This area is listed mainly in Book III, Chapters XI to XV concerning Europe and Book V and Chapter II concerning Asia. The Tabulae referred to these chapters are Tabula IX and X of Europe and Tabula I of Asia, with the largest part of modern Greece depicted in Tabula X (Fig. 1). In the areas of interest it is also included Cyprus, which is numerically described in Book V, Chapter XIII and is depicted in Tabula IV.

The World of Ptolemy is classified in Regions, since each Chapter is referred to one of them, giving by this way the concept of Atlas as it is understood today. The number of toponyms depicted on the Tabulae of Europe is high, compared to the other Regions of Asia and Africa. As it is also obvious from Fig. 1 the smaller the Tabula is the more important and detailed the region appears in Ptolemy’s *Geographia*.

![Figure 1. The ‘Tabulae’ in Ptolemy’s Geographia.](image)

As it is known, the importance of Ptolemy’s *Geographia* had “declined” for almost 10 centuries coming back in 13th c. thanks to Byzantine scholars in Constantinople influencing profoundly the revival of world’s geography and cartography in the Renaissance. This work is referenced mainly for the historic, artistic and editorial value of its editions and for the maps included but it is still almost unknown, with many questions left open, about the numerical part of its content.

**The positioning in Geographia**

The numerical description of the known World is done via the positioning of places on the spherical surface of the earth. In general, there are two fundamental ways of positioning (Fig. 2): the “coordinate mode”, which is based on the point placement with respect to two orthogonally intersected axes of reference, first set by Dichaearchus the 3rd c. B.C. and the “polar mode”, in which each point on a plane is determined by an angle and a dist-
tance from an origin. This second method recalls Aristotle’s formulation of the “directions of the wind” (3rd c. B.C.) which offers the method the angular component and the seaman’s skill in determining distances in sea.

![Image](image-url)

Figure 2. Two ways of positioning: (a) the polar mode based on Aristotle’s “directions of the wind” and seamen’s skill for estimation of distances and (b) the coordinate mode based on Dichaearchus

Ptolemy, in his *Geographia*, uses the Hypparchus “model”, for the “coordinate mode” point positioning on the surface of the sphere by using a pair of numbers, the geographical coordinates, which derive from the intersection of two orthogonal lines on the Earth-sphere, the parallels and the meridians. Each point is defined with two pure dimensionless numbers, *lambda* (λ) for the longitude of the point and *phi* (φ), for the latitude.

In this book, there are about ten thousand point positions on the globe, in pairs of geographic coordinates as they were known in Ptolemy’s days, which referred to geographic sites (i.e. towns, mountain picks, river mouths, promontories and other), identified or not with modern places. The coordinates, rounded-off in five minutes of arc, in both

![Image](image-url)

Figure 3. The origin of parallels and meridians in Ptolemy’s *Geographia*
orthogonal primer directions (parallels and meridians), are grouped according to the continental and regional classification followed by Ptolemy in his Geographia, following the known Ptolemaic reference system of parallels and meridians, the origin of which is respectively close to actual Equator and close to the Canary Islands almost 25 degrees west of the today’s origin at Greenwich (Fig.3).

Ptolemy Geographia and the Thessaloniki Project

Research on Ptolemy’s Geographia has started at the University of Thessaloniki, Greece, in the early Eighties, focused mainly, but not exclusively, on data related to territories which are now under the sovereignty of modern Greek state. This research is concentrated on the study and analysis of toponymy, in one hand, but more specifically on the intrinsic geometric structure given in Geographia, parameterized by the arrays of spherical geographic coordinates of longitude and latitude assigned to points on the earth surface apparently known in the times of Ptolemy.

The editions currently used\(^1\), are the following:

1. Vatopedion Codex (13\(^{th}\)-14\(^{th}\) century),
2. Marciana Codex (15\(^{th}\) century),
3. Codex Urbinas Lat.277, Biblioteca Apostolica Vaticana, 1472-1473 (Institut Cartogràfic de Catalunya, Barcelona),
4. Donnus Nicolaus Germanus mid-15\(^{th}\) century manuscript of Ptolemy’s Geographia as given in Codex Ebnerianus (Stevenson 1991: 92),
5. B.Pirckeymer’s edition, Lyon, 1535
6. G.Ruschelli’s edition, Venice 1574
8. two 19\(^{th}\) century editions by Nobbe (Leipzig 1843 printed edition, 1966),
9. Müller’s edition (Paris 1883),

In this phase of the Project the areas of interest are the regions of Thrace, Macedonia, Epirus, Achaia, Crete and Asia Minor. In these regions, almost 800 pairs of coordinates are listed, 600 of them referred to the actual territory of Greece. An important part of our research is also referred to Cyprus and to the Mediterranean coastline.

Processing the Ptolemy’s coordinates

According to the followed procedure in the Project, the coordinates for each place are first stored digitally in a database according to the currently available editions of Ptolemy’s Geographia. If necessary, the coordinates are transcribed from Byzantine Greek writing and before introducing into the database, they are visually checked on screen. The all around check for discrepancies in the point placement, especially the gross errors or misprints, are then detected and evaluated. The coordinate database for all Geographia ver-

\(^1\) The project is open for more editions.
After the proper coordinate auto-and cross-checking (an example shown in Fig. 5) and the correction from gross errors, they are projected onto a map with a relevant gnomonic projection from parallels and meridians, plotted in the same projection, using e.g. the elementary geometric projection ($r = R_0$, $x = R_0 \phi$), assuming a unit radius reference sphere ($R = 1$) for the earth's model. By this way, a map is constructed for each *Geographia* version plotted from the coordinates and depicting the toponyms of the regions (Fig. 6).
Figure 6. Coordinates from each *Geographia* version projected on a map: (a) Germanus; (b) Nobbe; (c) Mueller; (d) Vatopedion Codex; (e) Marciana Codex, (f) Ruschelli; (b) and (g) Greek / Latin edition by Mercator; (i) Pirckeymer; (j) Urbinas Codex Lat.277; (k) the new Bern edition (Basel, 2006).

Figure 7. Vector-wise visualization of the differences between Germanus and 3 other versions.
The point positioning visualized on screen makes easier the auto- and cross-checking of the coordinates, the detection of the differences, the gross errors, the double values and other displacements they may occur, as it is shown in Fig.7. Analysing the coordinates, an interesting point appeared concerning the relation between the given numerical coordinates of positions, as listed in Ptolemy’s *Geographia* and the same positions as they are graphically depicted in the later derived Ptolemaic maps. In this case the comparison can be done, only after the georeference of the maps to their proper coordinates (Livieratos 2006: 51-59). An example of this process is shown in Fig.8, where the georeference of de Turre’s (Rome 1490) Tabula X representation to the point-coordinates is performed in two ways: with respect to the coordinates given in *Geographia* and with respect to the geographic graticule (parallels and meridians). The comparison is done first as a point-wise process and second as a graticule-wise process.

![Figure 8](image)

**Figure 8.** Vector-wise visualization of the displacements of the points on de Turre’s Tabula X representation after its georeference to the coordinates of the points projected to a map. The georeference of the map is made by two ways: (a) using the *operational* coordinates as derived from the least coordinate discrepancy process and (b) using the graticule of the digital version of the map.

**Studies based on processing Ptolemy’s coordinates**

Apart from the coordinate analysis and comparisons based on the coordinates listed in the different *Geographia* versions, some geodetic methods and techniques are also applied and tested in order to analyse the coordinates from a geodetic point of view, since \( \lambda, \varphi \) generate spherical geometry, which is of geodetic interest. Some of the topics tackled in this issue are listed below followed by an illustrative example.

*The coordinate difference approach*

An innovative point in this Project is that the focus on the geographic coordinate issue is not so given to the coordinates of single points as much as to the coordinate-differences between the points. The interest is thus concentrated on the treatment of the relative position of the places introducing in geodetic analyses mostly the coordinate differences and the spherical shortest lengths (geodesics). The use of the coordinate-difference or in other words of the “relative positioning” in the analytic approach is a major contribution of the Project since was never tested in the past in any attempt to treat Ptolemy’s positioning.
With the coordinate difference approach the locality of the positioning properties are preserved opening new insights in the possible analyses using the Ptolemy’s coordinates, especially as the longitude differences are concerned. A key issue here is the implementation of the spherical approximation of the “direct” and “inverse” geodetic problems well-known in classical geodesy. In the “direct” case, given the coordinates \( \varphi, \lambda \) of one point, the length and the orientation of the shortest line (geodesic) interconnecting a second point it is possible the computation of the coordinates of the second point on a given sphere. In the “inverse” case, as it is the case in Geographia, two given pairs of \( \lambda, \varphi \) defining two points on the sphere is enough for the computation of the length and the orientation of the relevant interconnecting shortest line (geodesic) on a given sphere. In Fig. 9, are illustrated the length differences of the lines interconnecting the most distant points in the overall Geographia area treated in the Project. It can be seen the difference between the spherical length of the interconnecting line on a plane (ref: “distance”) and the plane geodesic counterpart.

![Figure 9. Geodesic lines and arc distances between points](image)

In Fig. 10 it is depicted the images of the geodesic interconnecting a central point with all other points in the overall area treated in the Project as they are computed from coordinates listed in a number of Geographia versions. This type of visualization offers a direct and comprehensive view of the differences and the similarities in the pattern of geodesics enriching relevant comparative and classification studies concerning the numerical content of the Geographia versions.
A part of the Project deals with the relation of geographic coordinates to the way they can be projected on a map. In this case, all the regions of Ptolemy’s *Geographia*, which are next to the Mediterranean sea are used, checked for errors, corrected and then projected in different ways to a map form. Some of these projections tested in that case, are those shown in Fig.11. This gives the basis for best-fitting (Boutoura and Livieratos 2006: 60-70) comparisons with relevant modern maps given in various cartographic projections and for deformation analyses of Ptolemy’s *Geographia* representations.

**Studies on the radius of Ptolemy’s sphere model(s)**

Another topic of the Project is the study on the radius of Ptolemy’s spherical model. Taking into account the fact that Ptolemy gives point positions on a sphere, it is possible through appropriate best fitting processes to find the closest sphere to these points, thus its corresponding radius. Coming closer to this problem, we found that this global approach is not actually the refined case. Looking to a regional group of points, it comes out that instead of a unique global sphere, numbers of regional best fitted spheres (Fig. 12) approach much better each region. This topic deserves very much a thorough investigation.
Figure 11. The Mediterranean coastline and the surrounding regions around it in map projections.

Figure 12. More than one spherical model fit best the coordinate-regions in Ptolemy’s Geographia.

A detailed research on the topic, as it was already shown for a test area (Livieratos 1998 / 2007) gives promising results on the spherical models used in calculating the geographic coordinates in the Ptolemy era.
**Consistency of Ptolemy $\lambda, \varphi$ with respect to Ptolemaic maps**

In the case of testing the consistency of Ptolemy given coordinates with those appeared on maps of Ptolemaic origin, it is carried out a comparison of the coordinates listed in the *Geographia* editions, referred to specific points, with the coordinates of the same points as depicted on the maps of the same edition. The latter is done with respect to the geographic graticule traced on the map. It is then easy to find the consistency between the given coordinates in the text with those derived from the relevant map. An example of this is shown in Fig.8, where comparing the numerical coordinates of the point positions as given in the *Geographia* text with the corresponding coordinates of the points on the georeferenced map, we found that there are deviations between the point position depicted and the relevant point according to the given coordinates. There are also deviations between the two georeferenced images which are derived from the best fitting process using as control points in the first case the point positions and in the other case the map graticule (Fig.13). From that work (Livieratos et al 2007) and a previous one (Livieratos 2006: 51-59), it can be concluded that the mapmaker followed distinct procedures for the drawing of the geographic graticule and for the plotting of the map content, as can be seen in the example of de Turre’s representation of Tabula X.

![Figure 13. The deviations between the two ways of georeferencing de Turre’s representation of Tabula X.](image-url)
Studies on the consistency of ‘Geographia’ λ,φ with respect to modern counterpart values

In this part of the research, it is studied the relation between Ptolemy’s geographical coordinates with their modern counterpart values. In order to perform such a comparison and to identify the possible coincidence of places in Ptolemy’s era with their today’s counterparts, it is important first, to compare the toponyms of each area with the toponyms of the corresponding area of actual territory of Greece, based mainly on relevant references with historical and archeological evidence.

Figure 14. Geographia toponymy on a modern map

In Fig. 14, it is shown on a modern map, the places where some of Ptolemy’s toponyms are detected according to historical, archaeological and other relevant evidence. These points play an important role since a set of them, properly distributed on the overall map area, is selected and brought into one to one correspondence with the actual coordinates of the same set of points in the modern map, after choosing a transformation system, involving a map projection and an earth model, as well. The result of the best fitting of Ptolemy’s coordinates to the modern counterparts is shown in Fig. 15.
Figure 15. Second order polynomial best fitting of Ptolemy’s representation into a modern map

Studies on $\lambda, \varphi$ differences between Ptolemy’s ‘Geographia’ and modern maps

Figure 16. The isolines of longitude differences, in degrees, between Ptolemy’s values and their actual counterparts
The research deals also with the order of magnitude of the longitude and latitude systematic differences of Ptolemy’s values from the today’s counterparts, both in broader and local scale. An example of this is the derived spatial distribution of the differences in longitude and latitude after the comparison of Ptolemy’s coordinates with their actual values, using the best fitting of Ptolemy’s representation to the modern map (Tsololini and Livieratos 2007). In Fig. 16 and Fig. 17 the pattern of the differences is illustrated, obviously of different order of magnitude in the terms of longitude and latitude.

Comparing coastlines from various ‘Geographia’ editions or other maps

Another topic is the comparison of the coastlines defined in different Geographia editions or in relevant maps, using geographical coordinates. The depiction of the results in this case, gives new and better views and insights on the correspondences, differences, alterations, corrections and changes, which are documented in the diachronic Geographia map versions. An example of this is shown in Fig.18, where the differences in the coastline of Crete are shown as derived from two different editions of Geographia, the Rome (de Turre, 1490) and the Utrecht (1695) editions (Livieratos 2006:51-59).
Figure 18. The island of Crete coastline in Rome (de Turre, 1490) and Utrecht (1695) Geographia editions.

Archaeological analysis of Ptolemy’s coordinates

In cooperation with archeologists and in addition to the geodetic analysis, the evidence offered from archaeological research and historical sources is used for the analysis of Ptolemy’s coordinates and their possible approximation to not yet identified sites of historical interest.

For the area studied in this Project, some of the ancient cities mentioned by Ptolemy, are known since either they exist today (Thessaloniki, Veroia etc.), or they are identified through important findings (e.g. inscriptions) during archaeological excavations (Dion, Pella etc.). However, many of Ptolemy’s ancient cities are still unidentified (Tristolos, Euporia etc.). In Fig. 19 these categories of ancient cities are shown on a modern map. One of the targets of this Project is the attempt to approximate the relative positions of Ptolemy’s unidentified cities by examining the positions of their neighbouring known locations. As an example, it is taken the ancient city of Aegae, the first capital of ancient Macedonia. In Fig.20, it is shown the Ptolemy’s positioning of Aegae (dark grey spot), on a modern map of the area, within a 10 km radius circle of uncertainty (light grey circle). An alternative positioning of Aegae (dark grey star) is according to a similarity best fitting of the Ptolemy’s site triangle “Edessa-Pella-Veroia” into its actual georeferenced counterpart. In the modern map it is shown that the closer archaeological site to this circle is that of Vergina (in red), only some ca. one to one and half kilometres far from this circle to North-East. The result confirms and amplifies the upshot by some experts, that Aegae is situated in the archaeological site of Vergina, the most probable place of Aegae (Manoledakis and Livieratos 2006: 31-41).
Figure 19. Known, identified and non-identified ancient cities and places from Ptolemy’s *Geographia* on a modern map.

Figure 20. The position of Ptolemy’s Aegae and the archaeological site of Vergina, only one to one and half kilometres away to North-East.
The gained experience working with archaeologists confirms that this kind of analysis is very useful in analysing the various hypotheses and scenarios in the process for the identification of important ancient sites and locations.

Concluding remarks

Ptolemy’s Geographia is a work, referenced mainly for the historic, artistic and editorial value of its editions and of the maps included in it, but it is still almost unknown for the numerical part of its content. The new processing methods and technologies, of great importance for the study of the geometric properties of early cartographic documents which are massively available today allow tackling this issue as well. The exploitation of geographical coordinates listed in Ptolemy’s Geographia is a good start in this domain. Comparative analyses of geographic coordinates between various editions are important to detect errors and discrepancies and to possibly derive a corrected list of coordinates for the places recorded by Ptolemy. Best fitting techniques are appropriate in order to compare early relevant cartographic representations with their modern counterparts. Geodetic analyses of the coordinates offer a new perspective for research and the approximation of Ptolemy’s toponyms with respect to their modern counterparts broadens the interest on the issue not only of cartographers but also of archaeologists who deal with the identification of archaeological sites on the ground confirming or rejecting relevant positioning hypotheses and scenarios.

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