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The Myth of the Square Chart

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Summary

The first nautical charts used to navigate in the Atlantic, in the late Middle Ages and Renaissance, were identical to the portolan charts of the Mediterranean, only differing in their geographic limits. With the introduction of astronomical navigation, and when a scale of latitudes was for the first time added to these charts, it quickly became obvious that their geometry had to be modified in accordance with the new navigational methods. The resulting hybrid model represented a major breakthrough in the nautical cartography of the Renaissance, marking the beginning of the evolution form the "maps based on routes", to the Ptolemy's system of geographical coordinates. Using cartometric analysis, and taking into account the navigational methods used in the 16th century, it will be shown that these charts were projectionless, in the sense that they were constructed plotting directly on the plane the observed latitudes, courses and distances, as if the Earth were flat.

Introduction

The genesis and evolution of nautical and terrestrial cartography in Europe are clearly distinct. When the first nautical charts appeared in the Mediterranean, probably during the 13th century, no scientific cartography had yet been born. Contrasting with the practical purpose and accuracy of the portolan charts, the existing maps were symbolic representations of an idealized world, strongly influenced by religion and mythology. The revolution caused by the translation and diffusion of Ptolemy's *Geography* in Europe, during the 15th century, had a relatively small impact in nautical cartography. And while terrestrial maps quickly began to adopt the model of the world and the new forms of representation suggested by Ptolemy, the concepts of geographical coordinates and map projections remained foreign to the portolan charts. Not because the mapmakers were ignorant of them (some made both types of maps) but due to the constraints imposed by the navigational methods of the time.

Since the appearance of the first portolan chart in the Mediterranean (the *Pisan chart* daes from about 1290), to the full adoption of the Mercator projection, in the middle of the 18th century, almost five centuries passed, during which the frontiers of the known world were dramatically expanded by the maritime discoveries. And still, the nautical charts used duing this large period continued to use the same apparently naïve methods of representation, based on routes (and later, on latitudes), in which the concept of cartographic projection was absent.

By the end of the Middle Ages, when the European sailors started to explore the open Atlantic Ocean, leaving behind the relative safety of the coastal waters, the method to fix the

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ship's position at sea, based on magnetic directions and estimated distances, was still the same as in the Mediterranean. However, with much larger periods of time without seeing land, this was not enough to guarantee a safe and effective navigation. As time elapsed since the last known position of the ship and no new navigational elements of information were obtained, the accuracy of the estimated positions quickly degraded to the point of becoming almost useless. By other words, it was quite easy for a ship to be lost at sea after only a few days of sailing, especially when the winds were not favourable and the pilots were forced to alter the course often.

The introduction of astronomical navigation by the Portuguese, in the middle of the 15th century, proved to be an adequate and durable solution for the problem. In the earliest period, the altitudes of a heavenly body (the Pole Star) were used only to estimate the north-south displacement of the ship relatively to some reference position, for instance, the port of Lisbon (see Barbosa, 1948:105; Albuquerque, 1991, vol. 2: 37-40; Albuquerque, 2001: 250-56). Later, in the second half of the 15th century, with the introduction of simple astronomical tables, it became possible to determine the latitude at sea, observing the Pole Star or the Sun with the quadrant and the astrolabe.

The first nautical charts used by the Italian, Portuguese and Spanish pilots to navigate in the Atlantic were identical to the portolan charts of the Mediterranean. The ship's position was determined as the intersection between a segment with origin in the last known (or estimated) position, in the direction of the magnetic course steered, and an arc of circumference whose radius was the distance estimated by the pilot. This method of fixing was known by the Portuguese as the *ponto de fantasia* (point of fantasy), a designation that emphasized the subjectivity of the estimation process. When the latitude started to be determined at sea, the method had to be modified in order to accommodate this new element of information. In general the three elements were not in perfect agreement, i.e., the observed latitude did not necessarily confirm the point of fantasy. In these cases, the latitude always prevailed over the course and the distance. The revised method, which will be explained later, was called the *ponto de esquadria* (set point).

Of course, this new way of determining the ship's position was not compatible with the existing charts, which had been constructed on the basis of magnetic directions and estimated distances. When a scale of latitudes was added to the old charts, it was soon recognized that their geometry had to be adapted to the new navigational methods[1]. The resulting hybrid model (the so-called *latitude chart*), is clearly distinct from the portolan type of chart, first of all because it is based on observed latitudes, but also due to its much larger extent, which makes the inconsistencies caused by the assumption of a flat Earth much larger too. In spite of its limitations, this innovation represented a major breakthrough in the nautical cartography of the Renaissance, marking the beginning of the evolution from the charts "based on routes" (or *route-enhancing maps*, according to Woodward, 1990) to Ptolemy's model, based on geographical coordinates (or *equipollent maps*, according to the same author). Due to the impossibility of determining the longitude at sea and the lack of knowledge on the spatial distribution of magnetic declination, this hy-

[1] The first known chart with a scale of latitudes dates from 1504. In 1514 the pilot João de Lisboa, in his *Tratado da Agulha de Marear* (Treaty of the Marine Compass), still complains about the discrepancies between the existing charts and the practise of navigation.

brid model would serve as a valuable navigational tool for more than two centuries, well beyond the appearance of the revolutionary Mercator projection, in 1569.

For a long time, it was accepted that the nautical charts of the 15th and 16th centuries were drawn according to the principles of the equidistant cylindrical projection (or equirectangular), with straight and equidistant meridians and parallels intersecting at right angles. Starting with a rectangular graticule (*rectangular chart*), when the latitudinal extent of the charted part of the Atlantic was relatively small, they would later evolve to a square graticule (*square chart*), when the Portuguese began to explore the tropical and equatorial regions. This thesis, whose origin is an erroneous interpretation of the chart's geometry by the 16th century pilots, was supported by prominent researchers of last century, like Pereira da Silva (1923: 62-67), Fontoura da Costa (1934: 199-208) and Armando Cortesão (1935: 43-70). Although the idea is not supported by cartometric analysis and does not take into account the old navigational methods, as António Barbosa (1938) has clearly shown a long time ago, it is still cited by important international specialists like John Snyder (1993: 6-8) and Mark Monmonier (2004: 28-29). One of the reasons is the fact that the work of Barbosa was written in Portuguese and didn't have the necessary international diffusion. Even in Portugal, his arguments were ignored or strongly contested by the authors of his time, having only been recognized by Pimentel (1984) almost fifty years later.

In this article it will be shown that: (i) the equirectangular projection is not, in general, adequate to marine navigation; (ii) the system of representation of the nautical charts of the 15th and 16th centuries, revealed by cartometric analysis, is clearly different from the equirectangular projection; (iii) the nautical charts of the 15th and 16th century were constructed using the so-called "planimetric method", plotting directly on the plane the latitudes, magnetic directions and distances observed at sea, as if the surface of the Earth were flat.

The equirectangular projection and marine navigation

The cylindrical equidistant projection, or equirectangular, is probably the simplest of all map projections: meridians and parallels are straight and equidistant, intersecting at right angles. The distances are conserved along all meridians and also on a chosen parallel of latitude: the standard parallel. If this parallel is the Equator, the graticule is square (*square chart*); otherwise, it is rectangular (*rectangular chart*).

The equirectangular projection is neither equivalent nor conformal. This means that, in general, the proportion of the areas measured on the surface of the Earth is not conserved and the scale varies with direction, distorting angles and shapes of small objects. Of these two properties only conformality is relevant for navigation, being necessary to the correct cartographic representation of courses and azimuths. In Figure 1 the various types of distortions affecting the equirectangular projection are illustrated using ellipses of distortion, here represented by wind roses from which sixteen rhumb lines (or loxodromes) irradiate. These distortions will now be analysed in detail, taking into consideration the needs of marine navigation:

– Angles: angles measured around a point are not generally conserved, causing the directions indicated by the wind roses to be wrong. It is one of the consequences of the variation of scale with direction; the other is the distortion of the circular shape of the wind roses. For example, the NE-SW direction indicated by the wind rose at 60º N, in the square chart, makes an angle of 60º with the meridian (instead of 45º). The only place where angles are conserved and the wind roses are circular is the Equator, in the square chart, and the Lisbon parallel (40ºN), in the rectangular chart.

- Distances: in both cases, distances are conserved in the direction of the meridians (this is reflected by the constant latitudinal extent of all wind roses). The scale along parallels varies with latitude, becoming equal to the scale along meridians only at the standard parallels: the Equator, in the square chart, and the parallel of Lisbon, in the rectangular chart.
- Directions: rhumb lines are not represented by straight lines, an important property for navigation. The curvature is small or moderate near the standard parallels, and between the Equator and the latitude of 30º, becoming much larger at latitudes greater than 60º.
- Areas: the proportion of areas measured on the Earth surface is not conserved in this projection. The fact is illustrated by the increasing size of the wind roses with latitude. However, this type of distortion is not relevant for navigation.

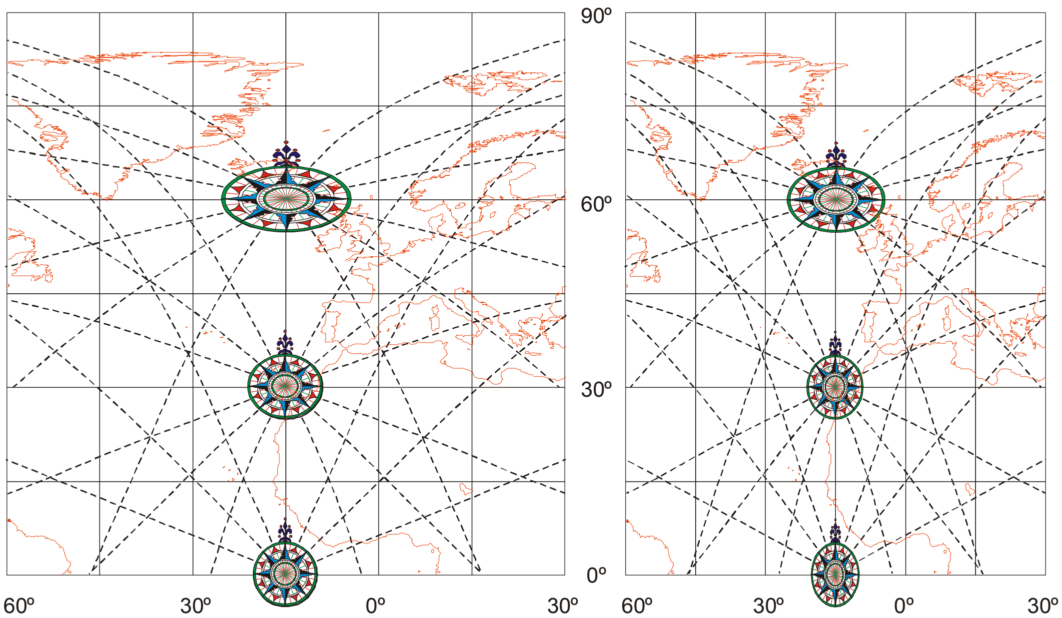

Figure 1. The distortions of the *square chart* (left) and the *rectangular chart* centred at Lisbon (right). The lines irradiating from the wind roses are loxodromes (rhumb lines).

Considering these distortions, could the equirectangular projection still be used for navigation? Because angles are not conserved and loxodromes are not represented by straight segments, it is not formally adequate for that purpose. To understand why not, imagine the difficulty in reading or plotting correctly a rhumb line between two places when that line is represented by a curve that makes variable angles with the meridians, or in measuring the corresponding distance on the chart when its linear scale varies with direction. However, if the area represented is small enough and the projection is correctly centred at its middle parallel, the distortions may be neglected (this is a general law applicable to most map projections, not only to the equirectangular). The idea is not new, as we will see next.

Pedro Nunes and the equirectangular projection

The Portuguese mathematician Pedro Nunes (1502-1578), who was the Major Cosmographer of the Kingdom from 1547 to his death, analyses the problems of the nautical charts used at the time in his *Tratado em defensam da carta de marear* (Treaty in defence of the nautical chart). After discussing the conception and use of the nautical charts, and recognizing their errors and limitations, he closes the subject with this text (Pedro Nunes, 2002: 141)[2]:

> *"But the best would be: to avoid all these troubles: to make the chart in many parts* [or sheets]*: with a good large scale: in which we keep the proportion of the meridian to the middle parallel: like Ptolemy does in the province tables: because all longitudes, latitudes and courses would be correct, at least there would not be a notable error: and carry the chart as a book* [...]. *And in the parts* [or sheets] *that do not contain land: that goes beyond eighteen degrees of latitude we can make all degrees equal to those of the meridian since the difference is small: and beyond* [the eighteen degrees of latitude]: *we will make the degrees of longitude equal to those of the middle parallel* [...]*"*

Pedro Nunes suggests the world map to be composed of many sheets, all of them in the equirectangular projection centred in their respective middle parallel. In addition to stating that a large scale should be used, he says nothing about the number of sheets or the scale in each of them. Two solutions are possible: either use the same principal scale for the whole chart, keeping the distance between parallels; or conserve the distance between meridians in order to keep the graphical continuity between adjacent sheets (Figure 2). Clearly, the first solution seems to be implicit in the text: "*and beyond* [the eighteen degrees of latitude] *we will make the degrees of longitude equal to those of the middle parallel*". That is also the opinion of the generality of the authors who commented the text, for example Pereira da Silva (1925: 203) and Fontoura da Costa (1934: 234-36). On the other hand it didn't seem to be the intention of Pedro Nunes to keep the graphical continuity between adjacent sheets since he explicitly refers to the construction of a book. More natural and intuitive would be - because it is closer to the geometry of the sphere - to conserve the scale along meridians, reducing the distance between them from sheet to sheet, solution that had the advantage of keeping constant the scale of leagues of the chart (which was deduced form the latitude scale).

[2] Translated by the author from the following Portuguese text: "*Mas o milhor seria: pera escusarmos todos estes trabalhos: que fizessemos a carta de muitos quarteyrões: de bom compasso grande: nos quais guardemos ha proporçao do meridiano ao parallelo do meio: como faz Ptolomeu nas tavoas das provincias: porque ficariam todas as longuras, alturas e rotas no certo, ao menos não avera erro notavel: e trazerse a carta em livro* [...]. *E nos quarteyrões em que não houver terra: que passe de desoyto graos daltura poderemos fazer todolos graos iguais aos do meridiano polla diferença ser pouca: e como daqui passar: faremos os graos da longura iguais aos do parallelo do meo* [...]"

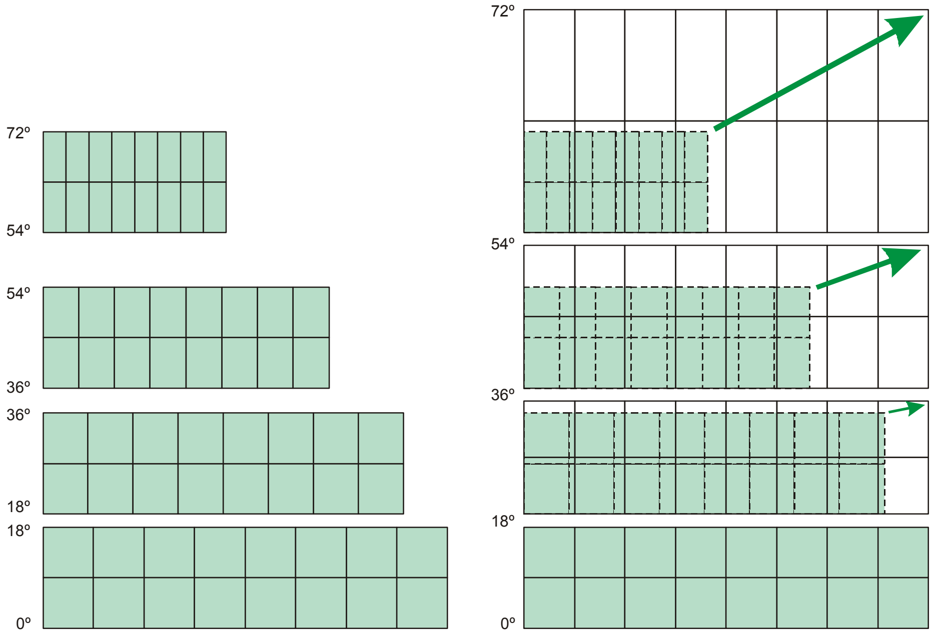

Figure 2. Pedro Nunes and the rectangular chart. Left, the version suggested by Pedro Nunes, with constant scale along meridians; right, the "Mercator type" solution, conserving the graphical continuity between meridians.

The second solution is, of course, only a little step from the Mercator projection, as Pereira Silva (1925: 203) has noted. But we don't think Pedro Nunes was prepared to take that step. However appealing the idea of associating his suggestion to the Mercator projection might be, the fact is the issue seems to be strange to the intentions of the mathematician. What he really wanted was to avoid the inconsistencies of the existing charts with a system of representation that could be considered, for practical purposes, conformal and with constant scale.

But was this suggestion of Pedro Nunes feasible? Apparently yes, since nothing really different from the usual forms of navigating seemed necessary: the pilots could continue to read and plot directly the latitudes, courses and distances on the chart, using the directions given by the wind roses and a single scale of leagues, avoiding the problems of inconsistency of the existing latitude charts. The discontinuity between adjacent sheets should not represent a serious difficulty since, as Gago Coutinho (1959: 156) noted, since dead reckoning by analytical processes was normally used. However, the impossibility of finding longitude at sea prevented the suggestion to be easily adopted, since relative positions in the east-west directions could not be correctly determined and represented, preventing the meridians to be drawn on the chart as straight and equidistant lines. More then three hundred years had yet to pass before this problem could be completely solved, with the invention of the maritime chronometer. The use of magnetic directions was, on the other hand, absolutely incompatible with any cartographic representation based on the "true" geographic North. Although the pilots were already able, in the first half of the 16th century, to recognize and measure magnetic declination, its spatial distribution had yet to be reflected in the charts. Note that the first Atlantic chart of isogonics was drawn only in 1701, by Edmund Halley. These same problems would contribute for the late adoption of the Mercator chart, presented in 1569, but fully accepted by the pilots only in the 18th century.

None of these questions was unknown by Pedro Nunes. That is why it is here considered that his suggestion had not the intention of being practical. If the mathematician really wanted to propose an operational solution for the inconsistencies of the existing model of chart, he would certainly explain his ideas in a clear and meticulous way, as was his habit. Contrasting with the long and exhaustive text in which the problems related to the conception and use of the sea chart are analysed, the present suggestion, only a few lines long, seems nothing more then a theoretical line of thinking. There is no historical evidence that his proposal had any practical influence on the way the charts were made, although Pedro Nunes, as the major cosmographer, had the authority and direct responsibility on the matter. Almost thirty years passed between the publication of this text (1537) and the *Petri Nonii Salaciensis Opera* (1566), in which no new ideas are introduced. In short, it seems clear that the author had no intention, in his Treaty in Defence of the Nautical Chart, of proposing the adoption of the equirectangular projection, because he was perfectly aware that the idea would not be easily put into practise due to the navigational and surveying problems involved.

Navigation and charting

The navigational methods of the 15^{th} and 16^{th} centuries were closely related to surveying and charting. The ways to fix the ship's position were very similar to those used to register the discovery of new lands and to plot them on the chart. In the earliest period, the positions were determined and registered on the basis of magnetic directions given by the compass and distances estimated by the pilots (*point of fantasy*); later, with the advent of astronomical navigation, the observed latitude became a preponderant element of information in the fixing process (*set point*).

In Figure 3 the *point of fantasy* and the *set point* are graphically defined. Because the charts were valuable tools (the graphite pencil had not been invented), the chart work was normally done with two pairs of dividers. The following rules, known as *emendas do ponto de esquadria* (amendments to the set point), set the procedure to determine the set point (Fontoura da Costa, 1983: 392-400):

- For courses less than 4 points (45º form North or South), the course prevails over the estimated distance: the set point is at the intersection of the parallel of observed latitude with line representing the magnetic course steered (case 1 in Figure 3);
- For courses more than 4 points, the estimated distance (D) prevails over the course: the set point is on the parallel of observed latitude at a distance D form the point of departure (case 2);
- For courses exactly equal to 4 points, the set point is at the intersection of the parallel of observed latitude with the perpendicular containing the point of fantasy (case 3);
- For E and W courses, the set point coincides with the point of fantasy.

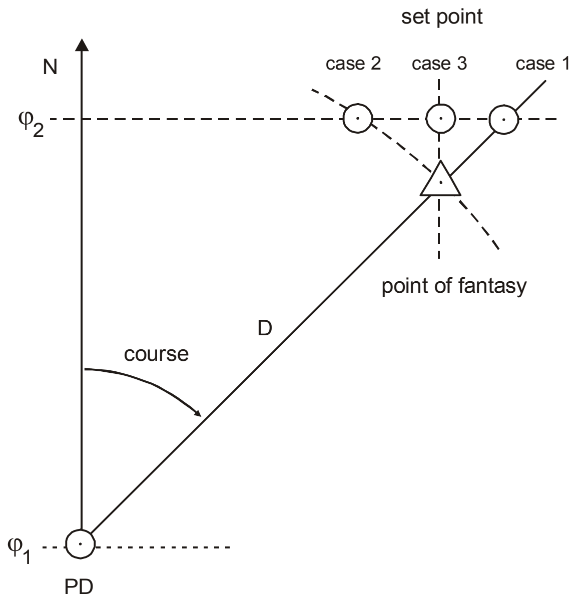

Figure 3. The *amendments to the set point*. The small triangle represents the *point of fantasy*, the circles represent the *set point*, PD is the point of departure and φ is the latitude (adapted form Fontoura da Costa, 1983).

We know, from the works of Pedro Nunes (1937: 127-141), Francisco da Costa (in Albuquerque, 1970) and Manuel Pimentel (1969: 137-142) how the charts were made. There is no evidence of the explicit use of any type of map projection in nautical cartography before 1569. On the contrary, all known historical sources are clear about the method employed to draw the charts, on the basis of observed latitudes, compass directions and estimated distances. Says Priest Francisco da Costa, who was Professor of the "Class of the Sphere" in the College of Saint Anton, Lisbon (in Albuquerque, 1970: 111):

> *"For representing the sea and show the land that confines with it in the hydrographic charts,* [...] *two things are presupposed, whose knowledge is absolutely necessary: the first* [is] *that* [...] *the heights of all ports, capes, inlets,* [...]*, etc., are known* [...]*; the second thing to be known by the hydrographer are the sailings of the coasts, ports, etc., both between each other and in respect to the same coast; we call sailing to a straight line or course that goes from one place to another, because these are the ways used to navigate in the sea,* [...]"[3]

Manuel Pimentel, when describing the three types of charts used at the time, in his *Arte de Navegar* (1969: 138), says:

> *"The second kind is of those charts called common or plane or of equal degrees, in which the meridians and parallels are represented by equidistant lines, which*

[3] Translated by the author from the text: "*Para nas cartas hidrográficas se representar o mar e dar mostra da terra que com ela confina* [...]*, se pressupõem duas coisas, cujo conhecimento é totalmente necessário: a primeira que se saibam* [...] *as alturas de todos os portos, cabos, enseadas,* [...], *etc.;* [...]; *a segunda coisa que há-de saber o hidrógrafo são as derrotas por que correm as costas, portos, etc., tanto entre si como em respeito da mesma costa; derrota chamamos a uma linha direita ou rumo que vai de um lugar a outro, que estes são os caminhos por onde o mar se navega,* [...]"

form equal squares, [...]. *These charts are made of courses and heights, putting the lands in their pole heights and courses that run with other lands,* [...]"[4]

Note how the cosmographer misinterprets the geometry of the so-called plane charts, as others have done before and after him. However, the method to "put the lands on the chart" according to their latitudes and courses to other lands is quite clear.

The planimetric method of charting

Suppose now that we use the method described by Francisco da Costa and Manuel Pimentel to plot the island of Terceira (in the archipelago of Azores) on a chart, using rhumb line courses and distances, with origin at Lisbon[5] (see also Gaspar, 2005). This can be done directly, with a single course R_1 and a distance D_1 between Lisbon and Terceira; or indirectly, following some other route. In Figure 4 (left) *A* represents Lisbon (φ = 39ºN), *B* the Madeira Island (φ = 32ºN) and *C* the Terceira island (φ = 39ºN). For simplicity, consider that the course between Lisbon and Madeira is SW, between Madeira and Terceira is NW and between Lisbon and Terceira is W. The rhumb line distances between these places are, respectively, 594, 594 and 803 nautical miles. The right part of Figure 4 shows three ways to plot the position of the island in a sheet of paper, using a constant scale: with a single track, *AC* (R_1 = W, D_1 = 803 nm); with two tracks, *ABC* (R_2 = SW, D_2 = 594 nm; R_3 = NW, D_3 = 594 nm); and with three tracks, *AA'C'C* (D_5 = 876 nm) and finally to North (D_5 = 420 nm). The corresponding charted positions of Terceira are C_1, C_2 and C_3, respectively, 803, 840 and 876 nautical miles from Lisbon. Note that the method conserves the north-south relative positions, so that the three points are in the same parallel.

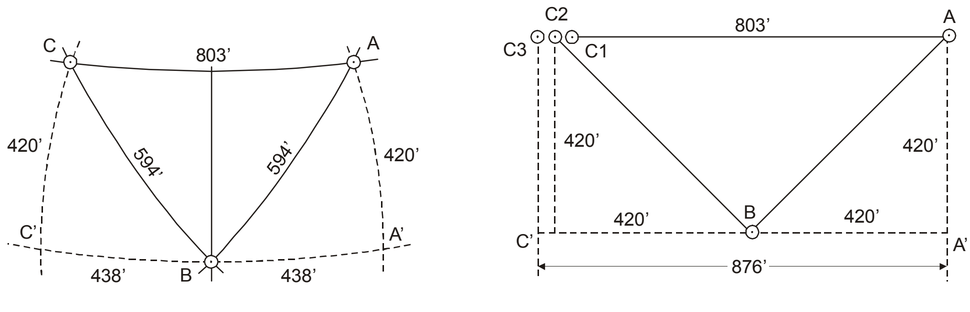

Figure 4. The inconsistency of the planimetric method. The position of point C (Terceira) was determined using three different tracks (AC, ABC and AA'C'C), plotting directly on the plane the angles and distances measured on the curved surface of the Earth.

These are relatively small differences if we take into account the crude navigational methods of the time. But when the planimetric method is used to represent large areas of

[4] Translated by the author from the text: "*A segunda espécie é daquelas cartas que se chamam comuns ou planas ou de graus iguais, nas quais os meridianos e paralelos se representam em linhas equidistantes que fazem quadrados iguais,* [...]. *Estas cartas se fazem por derrotas e alturas, pondo-se as terras nas suas alturas do pólo e nos rumos que se correm com outras terras,* [...]."

[5] A similar example was used by Pedro Nunes, in his *Treaty in Defence of the Nautical Chart* (1537), to show that the meridians could not be represented by straight and parallel lines.

the Earth surface, the inconsistencies that result from ignoring its curvature can be enormous and the resulting representations become strongly dependent on the tracks used to plot the places on the chart.

Consider now the representation of an area comprising the Mediterranean and part of the Atlantic and Indic Oceans, from the British Islands to the Cape of Good Hope and from the west coast of Brazil to Cape Guardafui, at the entrance of the Red Sea. Suppose that this new chart will be constructed on the basis of the set point method, using some typical maritime routes of the beginning of the 16th century. Finally, assume that no errors were made in the measurement of directions and distances at sea, so that the resulting distortions are only the consequences of using the planimetric method[6]. The magnetic declination will be assumed to be zero everywhere except in the Mediterranean, where a value of 8º E will be considered, and the *point of fantasy* method will be used[7].

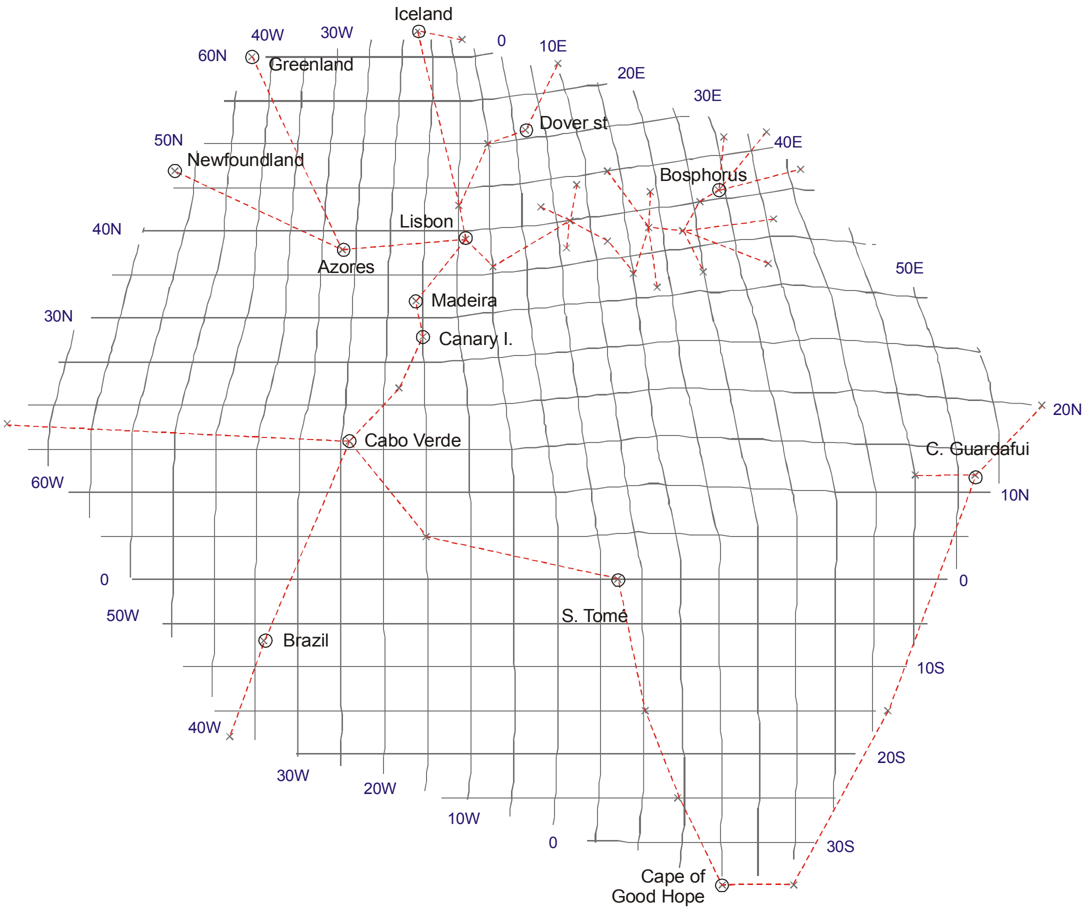

Figure 5. Geographical graticule that results from plotting some chosen rhumb line tracks on a plane, with constant scale. The little crosses represent the control points of the routes.

6 An alternative to this simple model is to use Multidimensional Scaling (MDS), generalized to spherical directions and distances, to "smooth" the inconsistencies of the planimetric method. This approach has been tried by author, with encouraging results.

7 The *point of fantasy* method continued to be used in the Mediterranean for a long time after the advent of astronomical navigation in the Atlantic. This is commented by Pedro Nunes in the cited work. The value of 8ºE for the magnetic declination is close to the NE tilt of the portolan charts of the 15th and 16th centuries.

Figure 5 shows the resulting geographical graticule, together with the routes used to implement the planimetric method. Since north-south distances are conserved, the parallels are represented by straight and equidistant lines, oriented in the east-west direction (except at, or near, the Mediterranean). The meridians are curves, making variable angles with the parallels. Also, the distance between adjacent meridians decreases with latitude, grossly conserving the convergence of meridians, and reflecting the use of a single distance scale.

In Figure 6 an excerpt of the Cantino chart is shown[8]. This is the oldest known nautical chart to show latitudes, implicitly represented by the tropics of Cancer and Capricorn, and by the Artic Circle. One of its intriguing distortions, which has been the object of various speculations, is the exaggerated east-west extension of Africa, causing the Isthmus of Suez to be enormous.

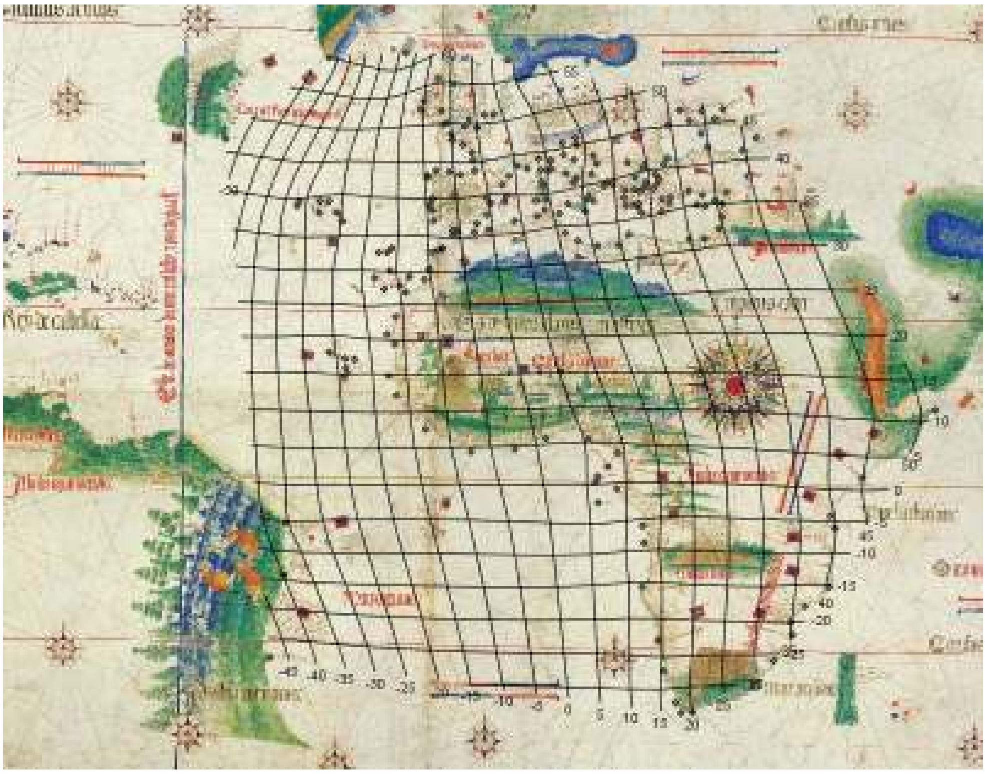

Figure 6. The geographical grid implicit to the Cantino map. Parallels and meridians are 5º spaced. The small circles represent the control points, used to interpolate the grid. Note the similarity with the graticule in Figure 5.

Using a sample of 200 places of known geographical coordinates positively identified in the old and a recent map, the geographic grid of meridians and parallels implicit to the representation was interpolated. The method was first suggested by Tobler (1966) and has been extensively used in the cartometric analysis of old maps, to identify the projections used, or the ones which are closest, and to assess cartographic errors. In the present case,

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[8] The Cantino chart was copied in Lisbon by an unknown cartographer, in 1502, from the chart standard deposited in *Casa da Mina*, and taken to Italy by Alberto Cantino, an agent of the Duke of Ferrara.

the computer application of Bernhard Jenny and Adrian Weber (Institute of Cartography, ETH Zurich), freely available trough the Internet (http://www.ika.ethz.ch/mapanalyst/index.html), was used to calculate the graticule.
The analysis of Figure 6 suggests the following remarks:

- The parallels are approximately straight, equidistant and oriented in the east-west direction, which is a direct consequence of using the *set point* method to draw the chart. The exception is the Mediterranean, where the *point of fantasy* method continued to be used after the advent of astronomical navigation;
- The accuracy of the charted latitude, evaluated by confronting the interpolated values with the representations of the Equator and tropical lines, is generally better than one degree, except in the northern part of the Atlantic;
- The meridians are curves, making variable angles with the parallels. The spacing between adjacent meridians decrease with latitude, grossly respecting the convergence of meridians, which is a consequence of using a single distance scale[9]. Another factor that certainly affects the orientation of the meridians is the non-corrected magnetic declination.
- There is a remarkable similarity between this graticule and the one in Figure 5. That seems to be a clear confirmation of the use of the planimetric method in the construction of the Cantino chart. The larger differences between both geometries are probably related to the non-corrected magnetic declination and other navigational errors.

The main reason for the enormous length of the Isthmus of Suez is the distortion caused by the use of the planimetric method, with a constant scale, as already noted by Teixeira da Mota (1977: 12). Other causes are the NE tilt of the Mediterranean and the incorrect orientation of the Red Sea, not directly surveyed by the Portuguese.
A more powerful tool than the simple model used above is Multidimensional Scaling (MDS), generalized to spherical directions and distances, as suggested by Tobler (1977). This approach allows the "smoothing" of the geometric inconsistencies resulting from the planimetric way of charting (in the real charts, this smoothing was an iterative process that took centuries), as well as the simulation of various navigational methods, under the influence of magnetic declination. Preliminary tests made by the author show a good agreement of the model results with the typical geometry of medieval portolan charts.

Concluding remarks

The idea that nautical charts of the 15th and 16th centuries were conceived in accordance with the geometrical principles of the equirectangular projection or, at least, that the charting methods automatically resulted in that type of representation, is rejected in this article. From the analysis done it can concluded, not only that the concept of map projection is foreign to the construction of those charts, but also that it is incompatible with it, a fact that was emphasized by Barbosa (1938, 1948). In particular, the thesis that the Portu-

[9] There is some similarity between this grid and the loximuthal projection. That would be indeed the exact result if a collection of rhumb line tracks were plotted from some chosen central position, as straight lines and with a constant scale.

guese nautical charts of that period were "square charts", supported by many researchers of last century, and still being cited by modern authors, is not confirmed by cartometric analysis and should be definitively withdrawn.
In this article it was shown that:

- The equidistant cylindrical projection (square or rectangular) is not suitable to marine navigation: angles are not conserved (it is not conformal) and rhumb lines are not straight segments. The idea of using large-scale equidistant cylindrical charts to keep distortions small, as suggested by Pedro Nunes, was incompatible with the use of magnetic directions and limited by the impossibility of determining longitude at sea;
- The system of representation of the nautical charts of the beginning of the 16th century, here revealed by a preliminary cartometric analysis of the Cantino chart, is clearly different from the equidistant cylindrical projection. In the so-called *latitude charts* parallels are straight, equidistant and oriented in the east-west direction, but meridians are curves that make variable angles with the parallels;
- The nautical charts of the 15th and 16th centuries were made using the planimetric method, by plotting directly on the plane the latitudes, magnetic directions and distances observed at sea, as if the Earth were flat. This is confirmed by the available historical sources and by the comparison between the results of the simple model implemented in this article and the geometry of the Cantino chart.

Even modern authors, when trying to fit exotic map projections to old nautical charts seem to neglect the fact, stated by Robinson (1985: 15) that "maps are to be looked at while charts are to be worked on". This also means that nautical charts, modern and old, are navigational tools designed in close agreement with actual navigational methods. To suggest that the portolan charts of the Middle Ages have some strange map projection background, or that the Portuguese nautical charts of the age of discoveries used the cylindrical equidistant projection, is to ignore the fact that they were both intended to support marine navigation, and that navigation was constrained by the use of magnetic directions and the impossibility of determining longitude. It should be noted that theses limitations affected, not only navigation at sea, but also the surveying operations.
A promising line of research is the use of Multidimensional Scaling, generalized to spheical distances and directions, to reconstruct the geometry of old nautical charts. This will allow the simulation of various navigational methods, under the influence of magnetic declination, as well as the smoothing of the inconsistencies.

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