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Some fundamentals for the study of the geometry of early maps by comparative methods

Keywords: Map comparison; History of maps; cartometric analysis; spatial fitting.

Summary
This paper explains the basic concepts and tools available for the comparison of early maps with modern counterparts. This comparison, which can be done using best fitting techniques through proper transformations of sets of points of the early map into corresponding sets of points of the modern counterpart, allows the study of the geometric and projective properties of early maps which is of main interest in the quantitative domain of the history of maps. Modern digital processing offers today easy tools for such comparative studies and this paper intends to give some fundamentals from the underlying theory and practice of the transformations, using no formulae but illustrations. In this way it is hoped that specialists in the field of cartographic heritage, including history of cartography and maps, having no previous opportunities to deal with the subject, could get hints on the issue, which may facilitate the dialogue with cartographers dealing with the nature of early maps and mapmaking.

Introduction

In mapping sciences and technologies, the general concept of 'comparison' is not new. It belongs to the fundamental methodological tools of mathematical physics and many applied sciences when it is impossible (or difficult) to effectuate direct geometric description (representation) of a given 'actual feature' without passing first through an intermediating process, which assists the geometric description. In this case the description is done indirectly by comparing the actual feature to an appropriate 'model', which is used as the intermediate in the representation process. For example, the use as 'models', of the mean sea level and the contours for the description of the 'actual feature' terrain relief in terms of heights, is an elementary example in topographic mapping. More complicated and elaborated examples are abundant in the specialized geodetic literature, even since the 18th and 19th centuries, where the comparison-concept is dominant in order to describe the validity of a 'model' in describing optimally an 'actual feature'. As an outcome of this comparison process is also the continuous improvement of the 'model' in order to fit better and better the 'actual feature' offering thus, better and better representations of its size and shape. The interface in the issue of comparison is the 'measurements'. They are performed on or from the 'actual feature' and are used in order to test the compliance and

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consistency of the 'model'. As a consequent result, the improvement of the 'model' can be obtained by applying a best-fitting process to the measurements.
This comparative process finds a series of applications in cartography (Boutoura and Livieratos 1986) and can be proved very efficient in the study of the geometric content and deformations of early maps, considered as 'models' using modern counterparts of known geometry in the place of the 'actual features'.
In this paper the fundamentals for possible comparative analysis of early maps are given. The focus is on a number of selected processes, which stand behind the comparison schemes, as used in the specialized cartographic literature on the issue. The paper, as addressed to non-specialists in the mathematics and technicalities of the process, stays intentionally out of the strict formulation of the transformations used. Instead, schematic illustrations for each case show the results making thus, more familiar the mathematics involved in the whole comparative analysis as it can be applied to the study of early map geometry.

The double-facet value of early maps

The above methodologies and techniques meet a lot of current applications in the modern representation schemes as applied in cultural heritage policies, management and engineering. The access, the documentation, the conservation, the preservation and the restoration of two-dimensional and three-dimensional monuments, works of art and artefacts, are all requiring properly elaborated representations as a fundamental precondition for any planning of study and intervention.
Cartographic heritage belongs rightfully to the overall cultural heritage (Livieratos 1999). Early maps, globes and atlases are a huge part not only of the artistic patrimony but also of the history of science and technology in the large. This characteristic of early maps in the context of the overall cultural heritage makes cartographic heritage double-facet. On one hand it deals with History and on the other with Science and Technology. Thus, the study of early maps is indeed worthy, not only as a thematic subject of History but also as a topic of the diachrony of mapping sciences and technologies.
In this perspective, the research on the geometric and projective parameterisation of early maps is of focal importance in understanding and even unveiling, in some cases, the origins and the dimensions of the mapping culture, knowledge, skill and technology available to the map-makers in their times. This importance is reflected, although scarcely and sparsely, in some excellent pieces of cartographic literature even since the late 19th century. However, this specific field of research is still rather rare in the overall literature of historic cartography and the question is whether or not this scarcity and sparseness proves that this research is of, say, secondary value or worthless. An easy or cursory response could be positive. But a close-up consideration of the whole issue may show that the lacunae in this domain of historic cartography enquiry are mainly because of the up to now technical and practical difficulties. Actually, the huge effort needed for proper data-acquisition and the heavy computational efforts required for such type of research, where patience is the very merit, made the venture extremely laborious and almost uninviting in the past. But today this shortcoming is no more valid because of the new computational

facilities available and because of a professional cartographic expertise, which is interested to extend its research activities to such matters.

Modern digital computational and graphic technologies, which are widely available today, embedded in unified digital processing environments, offer the right tools to face the acquisition shortcomings of the past and the huge computational labour. It offers a plurality of alternative schemes to approach the problem and to test a manifold of solutions giving access to powerful tools for evaluation and interpretation of the results. In this modern approach it is easy to proceed using the comparative method mentioned in the introduction, using a variety of computational channels, which are schematically presented here.

The comparative process

The comparative process for the study of early maps with respect to modern counterparts is posed as following: Let us having available a set A of control points lying on a plane-surface and a set B of corresponding points lying on another plane-surface (respectively, the 'actual feature' and the 'model' as stated above). The problem is then how to fit by a 'one-to-one' correspondence the set B into the set A getting a new set B' in which all points are optimally close (the closest possible) to the corresponding points of set A. In other words, one has to do here with the classical 'comparison issue', which is very common in geodetic sciences and technologies (see, e.g., Vaníček and Krakiwsky 1982) as it meets its best rationale in the comparison of the 'model' with the 'actual feature' with the 'measurements' as interface, a common scheme in the mapping sequence. The solution to this problem, in getting B', is given by the combination of the direct and inverse 'coordinate transformation' process, involving A and B. It is based on analytical geometry, well known in geodetic sciences as the 'best-fitting' process. However, the outcome implies inevitable alterations (deformations) in sizes and shapes of B as they are resulted in B' (Fig. 1).

The best-fitting transformation of B into A, from which B' is obtained, can be done by applying different schemes of relevant transformations, which are not only well documented in the affined geometric and geodetic literature but also operationally provided in a number of commercially available software packages dedicated to the coordinate transformation issue. Among those transformation schemes we mention here the simplest in use, namely the *similarity* (or conformal) transformation, the *affine* transformation, widely used in satellite image processing, the *projective* transformation, known in the rectification of central projections (the case of photography), the *polynomial* (usually the second order) transformation, known from the non-linear trend analysis in spatial statistics (thematic cartography) and the less used *finite element* transformation.

In order to be operational, each transformation scheme requires a different minimum number of control points with which the process can be carried out, i.e. 3 points in the similarity case, 4 points in the affine, 5 points in the projective, 7 points in the second order polynomial and at least 4 points in the finite element case. The output of the transformation, resulting the set B', is distributed 'globally' all over the plane surface and the final result, with all the expected alterations in sizes and shapes, is depending on the number of control points involved in the transformation process as well as on its proper spatial distribution on the plane-surface.

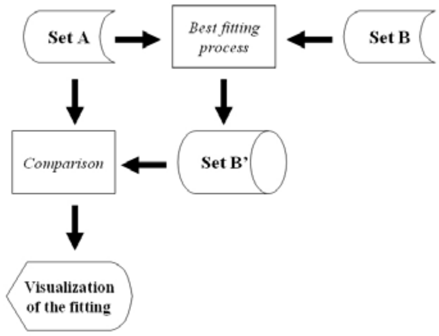

Figure 1. The comparison of sets A and B through a best fitting process

A particular kind of transformation, coming from animation technology, known as *morphing* (Guerra 1998, Gomes et al. 1999), can be also used due to its very interesting and useful best-fitting properties. On the contrary to the above mentioned, this type of transformation is applied locally around the control points used and, contrarily to the previous cases, is not necessarily affecting the areas distant from the control points.
From the list of transformation schemes mentioned here, the similarity, the affine, the projective and the polynomial transformation are not 'exact'. This means that comparing B' to A, the full coincidence of the corresponding sets of points is not fulfilled, in principle. On the contrary, in the case of the finite element and of the morphing transformation the coincidence is exact as far as the control points are concerned, namely the points used in the transformation process as 'common points'.

Testing the transformations

In order to illustrate the transformations used in a comparative study of B with respect to A, two corresponding plane sets of coordinates are used (Fig. 2). The equally sized circles on the surface of the set B are shapes deliberately drawn in order to assist the visualisation of the alterations in size and shape induced when transforming the set B into A. Two modes of transformations are used according to the number of control points involved in the process. In the first mode (the full-point mode) all points are taking part in the transformation process. In the second mode (the partial-point mode) selected sets of points are taken in different spatial distribution over the area in order to show how the transformation depends on the partial selection of the control points (in most cases it is mandatory in practice) and their spatial placement in the area affected by the transformation.

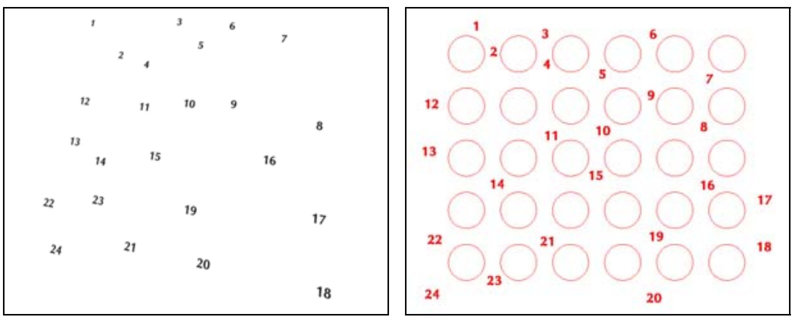

Figure 2. Left: The set of the control points A.
Right: The set of the corresponding points B with the equally sized circles.

The foul-point mode

In the foul-point mode all points are used to test the transformations (here, 24 arbitrary points are all taken in the process). In the following depictions alterations in size and shape are shown together with the field of point-displacements after the best fitting of B into A. The images of the displacements (right images in the Figs. 3 to 6) is the result of the comparison between the set A and the set B' of the best fitted points using all available points in both sets.
In Fig.3 is shown the *similarity* transformation B' (left) and the associated fitting to A (right). The linear result is a global rotation and a uniform scale change. The similarity (or conformality) is preserved as it can be shown from the unaltered shapes of the circles.

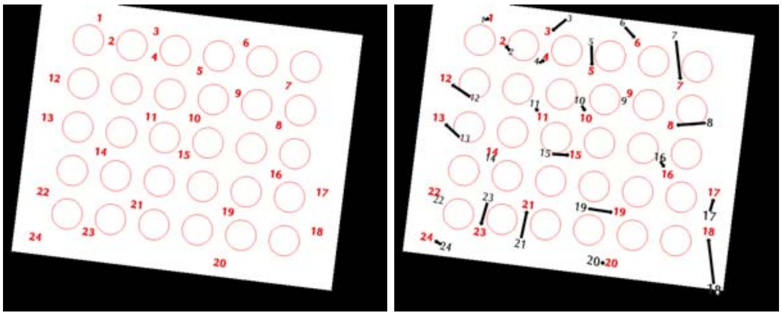

Figure 3. *Similarity* fit in the foul-point mode.
The shapes are kept unaltered (conformality) and the scale reduction is uniform all over the area.
In the right image the field of displacement are shown.

In Fig. 4 the results of the *affine* transformation is shown. The linear result is a global rotation and a shear angle and two uniform changes of scale in two intersecting directions as it evident from the change of shape of the circles into equal ellipses with the same axial orientation.

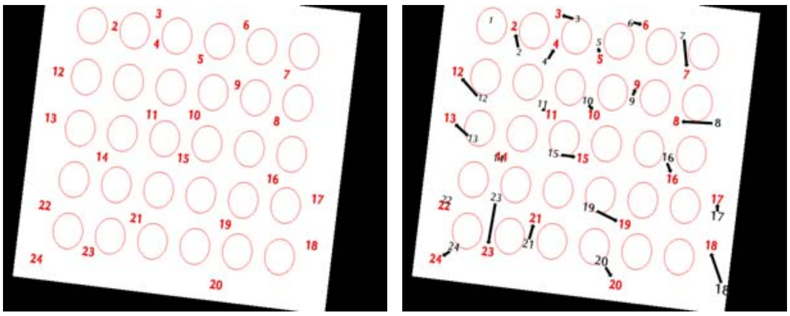

Figure 4. *Affine* fit in the foul-point mode.
The shapes and scale are changing along two intersecting directions.
In the right image the field of displacement are shown.

In Fig. 5 the *projective* case is illustrated. Here, the linear result is a perspective image with a non-uniform rotation and scale change of the size and shape of the circles, which transform into unequal ellipses, along strait lines converging to a focal point, with different axial orientations.

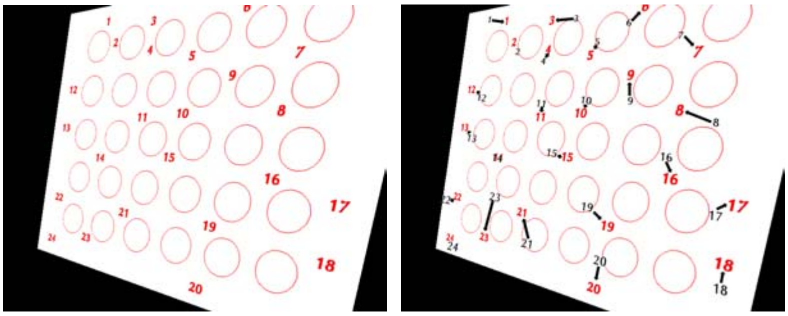

Figure 5. *Projective* fit in the foul-point mode.
Shapes and scale is changing along straight lines, which converge to a focal point.
In the right image the field of displacement are shown.

In Fig. 6, the second order *polynomial* fit is presented. The result is a curved image with a non-uniform, continuously changing rotation and scale change of the size and shape of the circles, which transform into unequal ellipses with different axial directions (tangent to the relevant curved directions). The effect of this non-linear transformation is covering the whole of the image and depends very much on the number of points and their spatial distribution of the test plane.

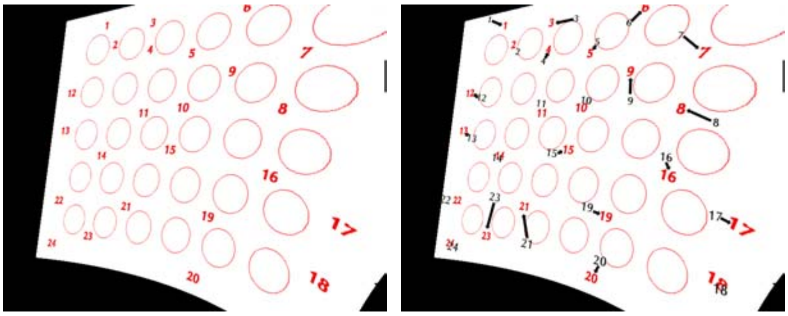

Figure 6. *Polynomial* (second order) fit in the foul-point mode.
The shapes and scale is changing along curved lines.
In the right image the field of displacement are shown.

In Fig. 7 the effect of the *finite element* and the *morphing* transformations is shown. Both preserve the exactness of the fitting, forcing the control points of both sets A and B’ to coincide, altering thus, in size and shape, the remaining space. The finite element method is applying a discrete step-wise areal operator in the process of transformation. This means that the greater are the areas of equal deformation (less control points used) the greater are the possible discontinuities of the representation of B’. The opposite (more control points) gives smoother results and an almost elastic behaviour of the fitted surface B’. Morphing is a very flexible ‘elastic’ transformation, which preserves the fitting at the control point and its close vicinity, leaving unaltered the rest of the area under transformation. It is a smooth process, which operates locally keeping intact the spatial continuity of B’.

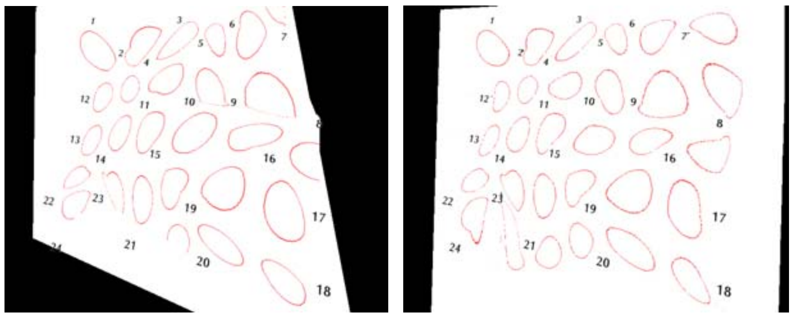

Figure 7. Left: The *finite element* fit in the full-point mode. The fitting is exact at the control points.
Shape discontinuities are evident in the spatial distribution of deformation,
which depends on the distribution of the control points.
Right: The *morphing* fit in the full-point mode. The fitting is exact at the control points.
Shape continuity in the spatial distribution of deformation is preserved.

The partial-point mode

As it is pointed out above, the minimum number of control points which is the sufficient and necessary condition in order to perform the transformation varies from 3, in the similarity case, to 7 in the second order polynomial case. In practice, of course, more points are taken in order to improve the solution depending always on the availability of control points and their spatial density and distribution. For the illustration of the effect of the minimum number of points involved in the transformation process and of the influence of the location of these points, three partial areas of the whole field are selected. The first is at the upper left part of the image, the second at the lower right and the third is covering almost the spatial totality of the field. In the following illustrations (Figs. 8 to 13) the effect of these partial fittings is shown for each transformation case, together with the point displacements and the areal alterations in size and shape.

In Fig. 8, the result of three local fits is shown using the 3 control points, which is the condition for the *similarity* transformation. The rotation, scale and shape properties of this transformation, is evident for all three cases of the control point location.

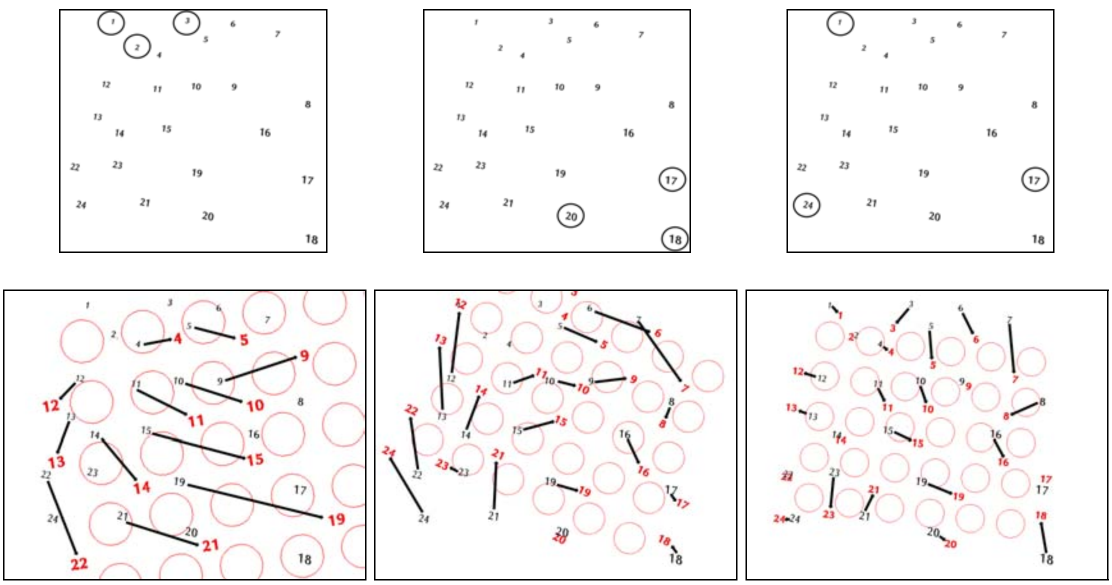

Figure 8. The *similarity* fitting using the minimum required 3 control points.

In Fig. 9, the result of the three local fits is shown using the required minimum 4 control points, which is the condition for the *affine* transformation. The rotation, scale and shape properties of this transformation are evident for all three cases of the control point location.

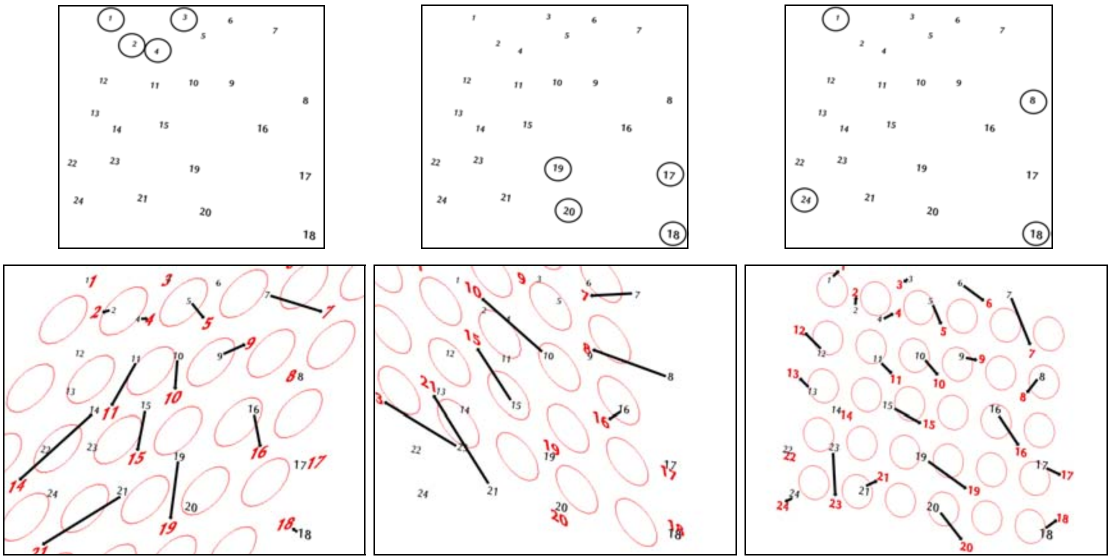

Figure 9. The *affine* fitting using the minimum required 4 control points.

In Fig. 10, the results of the three local fits for the *projective* transformation requiring minimum 5 control points, shows the difficulties in handling this type of fitting, which is suitable for specific applications when e.g. we know that the set B obeys certain conditions imposed by central (or focal) projections. This is the case of photography or of perspective plans.

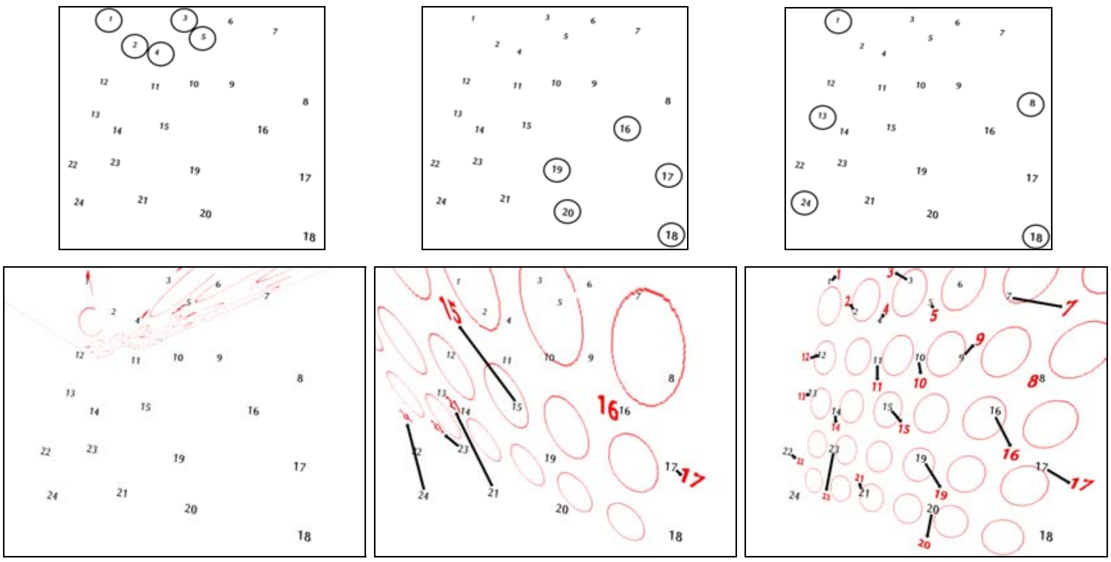

Figure 10. The *projective* fitting using the minimum required 5 control points.

In Fig. 11, the behaviour of another ‘uneasy’ type of transformation is shown. The polynomial fittings, among them the *second order polynomial*, should be used with care because if not properly treated tends to ‘explode’ the fitting as it can be seen in Fig. 10 (left), especially due to its non-linear properties. Of major importance here, is the good spatial distribution of the control points, which are taking part in the transformation. This transformation, requiring at least 7 control points, can be tested in cases we know that B suffers some non-lineal characteristics, as it is, e.g., the case of smoothly undulated surfaces.

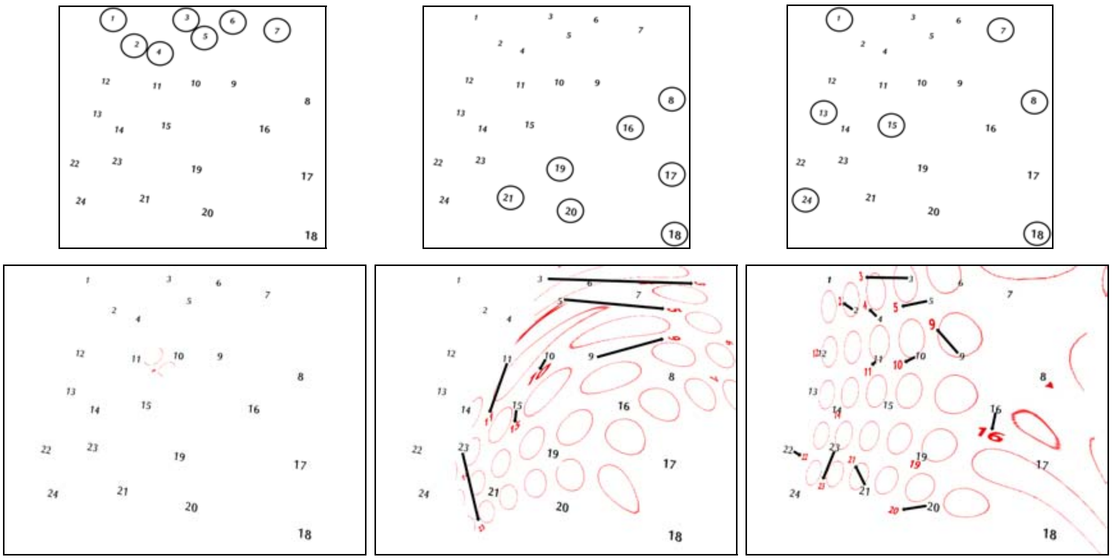

Figure 11. The *polynomial* (second order) fitting using the minimum required 7 control points.

In Fig. 12 the simulation concerns the case of *finite element* transformation. A minimum of 4 points is used here for illustrating the fitting and for the comparison of the affine fitting, which requires at least 4 control points. This is not certainly the case in the regular practice of finite element transformations, where the concern is to use a good number of control points properly distributed on the surface. The finite element fitting is a powerful tool in comparative studies if the user knows well how to implement it properly.

In Fig. 13 the results of a *morphing* fitting process is shown. The 4-point partial scheme is followed as in the finite element case, even if morphing (in principle and without any practical value) is operable with only one control point! The selection of the 4 points allows us to compare directly the morphing results with the results offered by the finite element method. The advantages in morphing are evident especially due to the very good behaviour in the local sense and the continuity and smoothness of the resulting image.

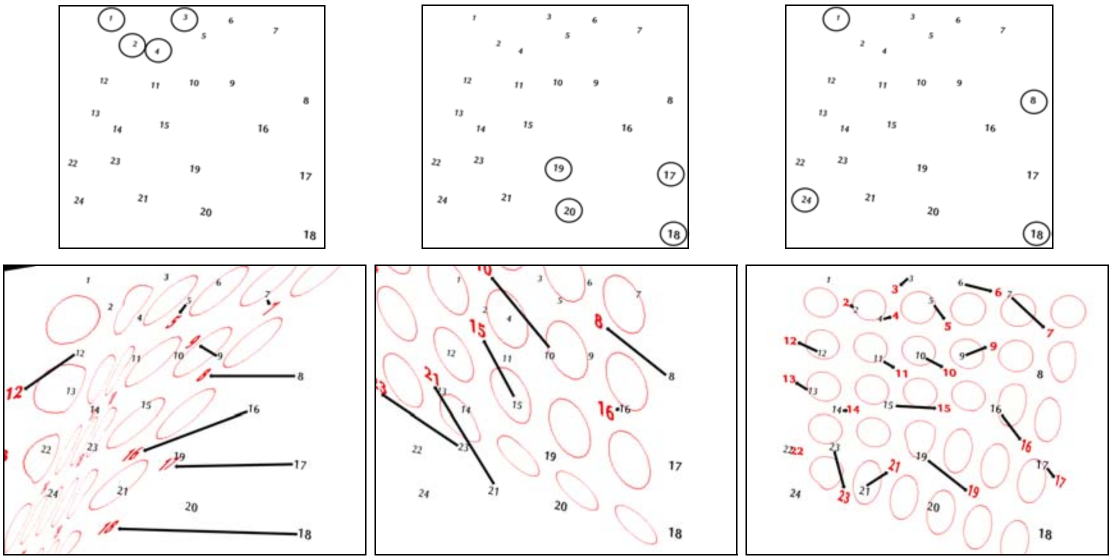

Figure 12. The *finite element* fitting using the minimum required 4 control points.

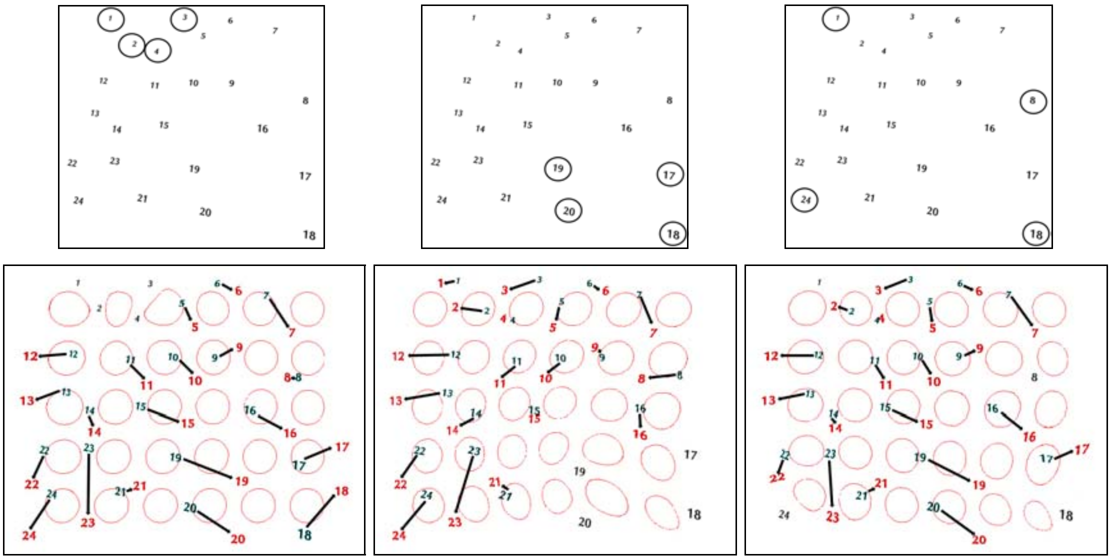

Figure 13. *Morphing* fitting with the use of 4 control points.

Concluding remarks

From the analysis illustrated above, in the simplest possible way, it is evident that the process of comparing early maps with modern counterparts is depending not only on the transformation scheme adopted but also on the density and on the spatial distribution of the control points taking part in the transformation.

The proper selection of the transformation depends on the problem the analyst is searching to approach and on the nature of the early map in use. For example, in the case that the comparison concerns the best fitting of the early map into a modern reference map, without deforming at all its original shape, the transformation to follow is *similarity*. This transformation allows only a global rotation of the early map and a possible global change in scale (which is not at all important in the comparison process). The global rotation is of major importance for the search of the orientation procedures used in the early times, es-

pecially in designing the nautical portolan maps. Similarity transformation is also applicable locally when the interest for the rotational properties of early maps is focused on certain parts of the map. In this case the best fit is local, without affecting, again, the global shape of the early map. In cases where the early maps, under a comparison process, are taken as copies of the originals, the use of other transformation schemes are recommended depending on the applied copying mechanism (see for a discussion, Daniil et al. 2003). These transformations are absorbing, in statistically optimal way, the alterations implied by the copying mechanism. When the interest is focused on the study of deformations, which are inherent in all early maps, then local transformations maybe the best tools for the analysis. In any case, the variety of options offered by the new digital technologies in analysing the geometric and projective properties of early maps gives the analyst the possibility to try and test a manifold of comparison strategies with many choices and alternatives in processing.
In this type of approach, the main concern is the proper experience of the analyst in such research, the good preparation in the cultural and technological context of early cartography and maps and the interaction and mutual understanding with the historian of cartography and map-making who could help offering valuable information and relevant directives.

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