Steps for identifying the projection of an old map

Keywords: Bessarabia map, Russian Empire map, map measurements, map projections.

Summary: Identifying the projection of a map poses significant challenges due to the wide variety of projections featuring similar properties. Correct projection identification requires the analysis of the network of meridians and parallels in terms of shape, then determining the type of curves through which they are represented so as to determine the projection group it belongs to (e.g. azimuthal, conic, cylindrical, etc.). In order to determine the class of cartographic projection (i.e. conformal, equal area, equidistant in certain directions), angle measurements must be made on the map along with measurements of meridian and parallel arcs for calculating the local scale along them. In the present work, the above steps were taken to identify the projection of two maps – one at a medium scale and the other at a small scale – namely the 1821 map of Bessarabia at the approximate scale of 1:1,100,000, and the 1827 general map of the Russian Empire with the Neighboring Polish Kingdom and Grand Duchy of Finland at the approximate scale 1:8,300,000. Both maps form part of the Geographical Atlas of the Russian Empire, the Kingdom of Poland, and the Grand Duchy of Finland, as compiled and engraved by Vasiliĭ Petrovich Piadyshev.

Introduction

The projection in which a map was made can give us information about the distortions produced in the on-plane representation of the respective zone. Thus, we can determine whether the territory is represented with distortions of its areas, lengths or angles, as well as the limits within which they vary. However, on many maps – especially older ones – the projection in which they were made has not been specified. In the present article, an analysis was made in order to deduce the projection in which the “General Map of the Russian Empire, the Neighboring Polish Empire and Grand Duchy of Finland”, as well as the “General Map of Bessarabia: Showing Postal and Major Roads” were drawn. These maps are included in the Geographical Atlas of the Russian Empire made by the lieutenant-colonel of the General Staff, Vasiliĭ Petrovich Piadyshev, an employee of the Military Topographic Depot. The atlas contains 60 maps of the governorates or provinces (gubernia) and regions (oblast) of the Russian Empire, featuring texts in Russian and French, as well as a list of abbreviations used on the maps. Although the year 1821 has been written on the cover of the atlas, the maps were actually made between 1820 and 1827, being presented in the atlas in chronological order with two exceptions (map number 44 – “Caucasus Province and Countries of Mountain Peoples” – 1825, and map number 59 – “Grand Duchy of Finland” – also 1825).

Materials and Methods

The steps to be followed to determine the map projection in which a map was drawn up, as proposed by multiple authors, including S. M. Soloviev (1955: 405-407), L. M. Bugayevskiy and J.
P. Snyder (1987: 246-247), and which were covered in the example outlined in this paper, are as follows:

- Determine the type of projection according to the general appearance of the meridian and parallel grid (azimuthal, cylindrical, conic projection, etc.);
- Determine the class in which the projection falls in terms of undistorted elements (equal area, conformal, equidistant along meridians or parallels, or arbitrary);
- Analyze if one of the meridians or parallels is represented by a straight line and if it is the axis of symmetry for the other meridians or parallels, respectively,
- Deduce whether the projection was made on a tangent or secant surface to the ellipsoid or sphere.

A very important observation made by the abovementioned authors is that in order to determine the projection of a map it must be drawn at a small scale, so that the appearance of the meridian and parallel grid can hint the type of projection used. For this very reason, we chose a small-scale map for the present study, namely the map of the Russian Empire, but also a medium-scale map, that of Bessarabia. Considering the dimensions of the printed maps and comparing the lengths of the meridian and parallel arcs measured on the maps with those calculated on the ellipsoid, it was deduced that the scale of the map of the Russian Empire is approximately 1:8,300,000, while the scale of the map of Bessarabia is about 1:1,100,000. One working hypothesis is that the Walbek ellipsoid was used in their making, the parameters of which were determined in 1819. Its semimajor axis is a=6376896 m, its semiminor axis, b=6355933 m and the flattening is f=1:302.8 (Soloviev 1955:13).

The present study was performed by going through the following steps for both of the analyzed maps:

**Step 1**
The scanned maps were firstly scaled. The maps contain only a bar scale whose unit of measurement is the verst, representing 1,067 km. Therefore, taking into account the length of a verst, the maps were scaled so that the graphical scale expressed the correct length.

**Step 2**
The images of the meridians and parallels were vectorized on the raster image and the appearance of the resulting curves was analyzed.

**Step 3**
The lengths of the meridians and parallels were measured so as to determine whether the map projection preserved certain meridians and/or parallels undistorted. For this purpose, in order for the results to be as concluding as possible, arcs as short as possible should be measured.

**The Russian Empire Map**

The full title of analyzed map is: “General Map of the Russian Empire and the Neighboring Polish Empire and Grand Duchy of Finland: With the Distance in Versts on Postal Roads between Provincial Cities; and in Boundary Provinces from the Provincial City to County Towns and Foreign Borders; with a Table of Distance in Versts between 73 Notable Cities”. The area represented on the map extends from 35° to 75° on latitude and from 15° to 215° on longitude. As it is written in the lower-left corner of the map, the longitudes were measured from the meridian

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1 The translation of the map title can be find at: https://www.loc.gov/resource/gdclccn.2018688695/?r=-0.511,-0.284,1.723,0.753,0, last accessed in May 2022.
with the longitude 20° west of Paris, meaning that the prime meridian was the Ferro meridian. The longitudes ranged from 0° to 360°.

The map of the Russian Empire consists of six map sheets glued together, which were marked 60 a, 60 b, 60 c, as can be seen in the upper-right corner on the first row, and we assume that analogously on the second row were marked 60 d, 60 e and 60 f, yet this is not visible on the map itself. The dimensions of a map sheet are equal to the dimensions of the other maps in the atlas represented on a single sheet, namely 40 x 38 cm, thereby resulting in the map of the Empire measuring dimensions of 120 x 78 cm.

**Step 1**
The map of the Russian Empire was scaled based on the graphical scale map, its bar scale being divided from 100 to 100 versts.

**Step 2**
The images of the meridians and parallels were vectorized on the raster image. It can be seen that the images of the meridians form segments of concurrent lines (the distances between the points of intersection vary between 10 and 20 mm), while the images of the parallels are circles with the distances between their centers varying from 0.5 to 6 mm.

**Step 3**
The lengths of the meridians were measured on the map in order to determine whether the map projection was equidistant along the meridians. The values obtained are shown here in Table 1 where the following notations were used:

\[
\begin{align*}
    s_m (1^\circ) &= \text{the length of } 1^\circ \text{ meridian arc} \\
    \Delta s_m &= s_m \text{ calculated } - s_m \text{ measured} \\
    n &= \text{denominator of the map scale}
\end{align*}
\]

The lengths of the parallels were measured so as to determine whether the projection preserved certain parallels undistorted. For this purpose, in order for the results to be as concluding as possible, and assuming that the measurements were made on a printed map, then arcs as short as possible should be measured, therefore both the meridians and the parallels were divided into 1° arcs. They were compared with the calculated values assuming that the Earth is a sphere with a radius of 6362884.283 m (which is the average radius of curvature of the Walbek ellipsoid for 55° latitude). The values obtained are depicted below in Table 2 where the following notations were used:

\[
\begin{align*}
    s_p (1^\circ) &= \text{the length of } 1^\circ \text{ parallel arc} \\
    \Delta s_p &= s_p \text{ calculated } - s_p \text{ measured} \\
    n &= \text{denominator of the map scale}
\end{align*}
\]

In order to determine whether the map projection was conformal, several angles were measured on the Russian Empire map between meridians and parallels at the graticule nodes. The values obtained are presented here in Table 3.
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<th>λ</th>
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<th>Δ sm [m]</th>
<th>Δ sm/n [mm]</th>
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Table 1: Meridian arcs measured and calculated, including the differences between them (Earth sphere).
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<th>$\varphi$</th>
<th>Sp ($1^\circ$) measured</th>
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<td>60°</td>
<td>57372.00</td>
<td></td>
<td>205.36</td>
<td>0.02</td>
<td>e</td>
</tr>
<tr>
<td>60°-65°</td>
<td>55°</td>
<td>32083.04</td>
<td>28742.7</td>
<td>3340.34</td>
<td>0.40</td>
<td>b</td>
</tr>
<tr>
<td>80°-85°</td>
<td>55°</td>
<td>31524.21</td>
<td></td>
<td>2781.50</td>
<td>0.34</td>
<td>b</td>
</tr>
<tr>
<td>95°-100°</td>
<td>55°</td>
<td>32374.23</td>
<td></td>
<td>3631.52</td>
<td>0.44</td>
<td>b</td>
</tr>
<tr>
<td>110°-105°</td>
<td>55°</td>
<td>31741.86</td>
<td></td>
<td>2999.15</td>
<td>0.36</td>
<td>b</td>
</tr>
<tr>
<td>125°-130°</td>
<td>55°</td>
<td>30833.84</td>
<td></td>
<td>2091.14</td>
<td>0.25</td>
<td>b</td>
</tr>
<tr>
<td>140°-145°</td>
<td>55°</td>
<td>31777.08</td>
<td></td>
<td>3034.37</td>
<td>0.37</td>
<td>b</td>
</tr>
<tr>
<td>175°-180°</td>
<td>55°</td>
<td>31459.75</td>
<td></td>
<td>2717.04</td>
<td>0.33</td>
<td>e</td>
</tr>
<tr>
<td>190°-195°</td>
<td>55°</td>
<td>32455.71</td>
<td></td>
<td>3713.00</td>
<td>0.45</td>
<td>e</td>
</tr>
<tr>
<td>30°-35°</td>
<td>55°</td>
<td>32150.23</td>
<td></td>
<td>3407.53</td>
<td>0.41</td>
<td>a</td>
</tr>
</tbody>
</table>

Table 2: Parallel arcs measured and calculated, including the differences between them (Earth sphere).
The “General Map of Bessarabia: Showing Postal and Major Roads” was drawn on a sheet measuring 40 x 38 cm.² The area represented on map extends from 45°00’ to 48°48’ on latitude and from 44°00’ to 49°00’ on longitude. As in the case of Russian Empire map, longitudes are measured from the 20° west of Paris meridian.

**Step 1**
The map was scaled based on its bar scale, which is divided from 10 to 10 versts.

**Step 2**
The images of the meridians and parallels were vectorized on the raster image. It can be seen that the images of the meridians are segments of concurrent lines (the distances between the points of intersection vary between 6 mm and 74 mm), while the images of the parallels form circles with the distances between their centers varying from 55 mm to 239 mm.

**Step 3**
The lengths of the meridians, divided into arcs of 10’ were measured on the map in order to determine whether the map projection was equidistant along the meridians. The values obtained are shown here in Table 4 for the hypothesis Earth – Walbek ellipsoid and in Table 5 for the hypothesis Earth sphere with a radius of 6361450.582 m (the mean radius of the Walbek ellipsoid for the 47° latitude).

The lengths of the parallels, also divided into arcs of 10’, were measured so as to determine whether the projection preserved certain parallels undistorted. The values obtained are shown in Table 6 for the hypothesis Earth – Walbek ellipsoid and in Table 7 for the hypothesis Earth – sphere having the same characteristics mentioned above. In tables 6 and 7 were used the following notations:

\[
\begin{align*}
\text{s}_m (10') &= \text{the length of 10’ meridian arc} \\
\Delta s_m &= \text{s}_m \text{ calculated} - \text{s}_m \text{ measured} \\
S_p (10') &= \text{the length of 10’ parallel arc}
\end{align*}
\]

²https://www.loc.gov/item/2018688650/
Δs_p = s_p calculated - s_p measured
n = denominator of the map scale

In order to determine whether the map projection was conformal, several angles were measured on the Bessarabia map between meridians and parallels at the graticule nodes (See Table 8)

<table>
<thead>
<tr>
<th>λ</th>
<th>Limits of the arc</th>
<th>s_m (10') measured [m]</th>
<th>s_m (10') calculated [m]</th>
<th>Δs_m [m]</th>
<th>Δs_m/n [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>44°</td>
<td>46°-46°10'</td>
<td>18447.60</td>
<td>18522.06</td>
<td>74.46</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>47°-47°10'</td>
<td>18418.31</td>
<td>18525.27</td>
<td>106.96</td>
<td>0.10</td>
</tr>
<tr>
<td>45°</td>
<td>46°-46°10'</td>
<td>18437.24</td>
<td>18522.06</td>
<td>84.82</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>47°-47°10'</td>
<td>18438.26</td>
<td>18525.27</td>
<td>87.01</td>
<td>0.08</td>
</tr>
<tr>
<td>46°</td>
<td>46°-46°10'</td>
<td>18322.07</td>
<td>18522.06</td>
<td>199.99</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>47°-47°10'</td>
<td>18457.73</td>
<td>18525.27</td>
<td>67.54</td>
<td>0.06</td>
</tr>
<tr>
<td>47°</td>
<td>46°-46°10'</td>
<td>18345.48</td>
<td>18522.06</td>
<td>176.58</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>47°-47°10'</td>
<td>18448.14</td>
<td>18525.27</td>
<td>77.12</td>
<td>0.07</td>
</tr>
<tr>
<td>48°</td>
<td>46°-46°10'</td>
<td>18349.01</td>
<td>18522.06</td>
<td>173.05</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>47°-47°10'</td>
<td>18459.80</td>
<td>18525.27</td>
<td>65.46</td>
<td>0.06</td>
</tr>
<tr>
<td>49°</td>
<td>46°-46°10'</td>
<td>18379.86</td>
<td>18522.06</td>
<td>142.20</td>
<td>0.13</td>
</tr>
<tr>
<td></td>
<td>47°-47°10'</td>
<td>18466.63</td>
<td>18525.27</td>
<td>58.64</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 4: Meridian arcs measured and calculated, including the differences between them (Walbek ellipsoid).

<table>
<thead>
<tr>
<th>λ</th>
<th>Limits of the arc</th>
<th>s_m (10') measured [m]</th>
<th>s_m (10') calculated [m]</th>
<th>Δs_m [m]</th>
<th>Δs_m/n [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>44°</td>
<td>46°-46°10'</td>
<td>18447.60</td>
<td>18524.71</td>
<td>57.11</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>47°-47°10'</td>
<td>18418.31</td>
<td>18525.27</td>
<td>86.40</td>
<td>0.08</td>
</tr>
<tr>
<td>45°</td>
<td>46°-46°10'</td>
<td>18437.24</td>
<td>18524.71</td>
<td>67.47</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>47°-47°10'</td>
<td>18438.26</td>
<td>18525.27</td>
<td>66.45</td>
<td>0.06</td>
</tr>
<tr>
<td>46°</td>
<td>46°-46°10'</td>
<td>18322.07</td>
<td>18524.71</td>
<td>182.64</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>47°-47°10'</td>
<td>18457.73</td>
<td>18524.71</td>
<td>46.98</td>
<td>0.04</td>
</tr>
<tr>
<td>47°</td>
<td>46°-46°10'</td>
<td>18345.48</td>
<td>18524.71</td>
<td>159.23</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>47°-47°10'</td>
<td>18448.14</td>
<td>18524.71</td>
<td>56.54</td>
<td>0.05</td>
</tr>
<tr>
<td>48°</td>
<td>46°-46°10'</td>
<td>18349.01</td>
<td>18524.71</td>
<td>155.70</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>47°-47°10'</td>
<td>18459.80</td>
<td>18524.71</td>
<td>44.91</td>
<td>0.04</td>
</tr>
<tr>
<td>49°</td>
<td>46°-46°10'</td>
<td>18379.86</td>
<td>18524.71</td>
<td>124.85</td>
<td>0.11</td>
</tr>
<tr>
<td></td>
<td>47°-47°10'</td>
<td>18466.63</td>
<td>18524.71</td>
<td>38.08</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Table 5: Meridian arcs measured and calculated, including the differences between them (Earth sphere R=6361450.582m).
<table>
<thead>
<tr>
<th>λ</th>
<th>φ</th>
<th>( s_p ) (10') measured [m]</th>
<th>( s_p ) (10') calculated [m]</th>
<th>( \Delta s_p ) [m]</th>
<th>( \Delta s_p/n ) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>44°-44°10'</td>
<td>46°</td>
<td>13124.32</td>
<td>12907.70</td>
<td>-216.62</td>
<td>-0.20</td>
</tr>
<tr>
<td></td>
<td>47°</td>
<td>12608.07</td>
<td>12673.20</td>
<td>65.13</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>48°</td>
<td>12314.09</td>
<td>12434.80</td>
<td>120.71</td>
<td>0.11</td>
</tr>
<tr>
<td>45°-45°10'</td>
<td>46°</td>
<td>13131.33</td>
<td>12907.71</td>
<td>-223.62</td>
<td>-0.20</td>
</tr>
<tr>
<td></td>
<td>47°</td>
<td>12785.95</td>
<td>12673.20</td>
<td>-112.75</td>
<td>-0.10</td>
</tr>
<tr>
<td></td>
<td>48°</td>
<td>12336.94</td>
<td>12434.80</td>
<td>97.86</td>
<td>0.09</td>
</tr>
<tr>
<td>46°-46°10'</td>
<td>46°</td>
<td>13088.77</td>
<td>12907.71</td>
<td>-181.06</td>
<td>-0.16</td>
</tr>
<tr>
<td></td>
<td>47°</td>
<td>12651.62</td>
<td>12673.20</td>
<td>21.58</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>48°</td>
<td>12250.79</td>
<td>12434.80</td>
<td>184.01</td>
<td>0.17</td>
</tr>
<tr>
<td>47°-47°10'</td>
<td>46°</td>
<td>13134.61</td>
<td>12907.71</td>
<td>-226.9</td>
<td>-0.21</td>
</tr>
<tr>
<td></td>
<td>47°</td>
<td>12673.23</td>
<td>12673.20</td>
<td>-0.03</td>
<td>0.00</td>
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<tr>
<td></td>
<td>48°</td>
<td>12244.63</td>
<td>12434.80</td>
<td>190.17</td>
<td>0.17</td>
</tr>
<tr>
<td>48°-48°10'</td>
<td>46°</td>
<td>12897.09</td>
<td>12907.71</td>
<td>10.62</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>47°</td>
<td>12591.21</td>
<td>12673.20</td>
<td>81.99</td>
<td>0.07</td>
</tr>
<tr>
<td></td>
<td>48°</td>
<td>12229.64</td>
<td>12434.80</td>
<td>205.16</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table 6: Parallel arcs measured and calculated, including the differences between them (Walbek ellipsoid).

<table>
<thead>
<tr>
<th>λ</th>
<th>φ</th>
<th>( s_p ) (10') measured [m]</th>
<th>( s_p ) (10') calculated [m]</th>
<th>( \Delta s_p ) [m]</th>
<th>( \Delta s_p/n ) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>44°-44°10'</td>
<td>46°</td>
<td>13124.32</td>
<td>12854.45</td>
<td>-269.87</td>
<td>-0.25</td>
</tr>
<tr>
<td></td>
<td>47°</td>
<td>12608.07</td>
<td>12620.18</td>
<td>12.11</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>48°</td>
<td>12314.09</td>
<td>12382.07</td>
<td>67.98</td>
<td>0.06</td>
</tr>
<tr>
<td>45°-45°10'</td>
<td>46°</td>
<td>13131.33</td>
<td>12854.45</td>
<td>-276.88</td>
<td>-0.25</td>
</tr>
<tr>
<td></td>
<td>47°</td>
<td>12785.95</td>
<td>12620.18</td>
<td>-165.77</td>
<td>-0.15</td>
</tr>
<tr>
<td></td>
<td>48°</td>
<td>12336.94</td>
<td>12382.07</td>
<td>45.13</td>
<td>0.04</td>
</tr>
<tr>
<td>46°-46°10'</td>
<td>46°</td>
<td>13088.77</td>
<td>12854.45</td>
<td>-234.32</td>
<td>-0.21</td>
</tr>
<tr>
<td></td>
<td>47°</td>
<td>12651.62</td>
<td>12620.18</td>
<td>-31.44</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td>48°</td>
<td>12250.79</td>
<td>12382.07</td>
<td>131.28</td>
<td>0.12</td>
</tr>
<tr>
<td>47°-47°10'</td>
<td>46°</td>
<td>13134.61</td>
<td>12854.45</td>
<td>-280.16</td>
<td>-0.25</td>
</tr>
<tr>
<td></td>
<td>47°</td>
<td>12673.23</td>
<td>12620.18</td>
<td>-53.05</td>
<td>-0.05</td>
</tr>
<tr>
<td></td>
<td>48°</td>
<td>12244.63</td>
<td>12382.07</td>
<td>137.44</td>
<td>0.12</td>
</tr>
<tr>
<td>48°-48°10'</td>
<td>46°</td>
<td>12897.09</td>
<td>12854.45</td>
<td>-42.64</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>47°</td>
<td>12591.21</td>
<td>12620.18</td>
<td>28.97</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>48°</td>
<td>12229.64</td>
<td>12382.07</td>
<td>152.43</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Table 7: Parallel arcs measured and calculated, including the differences between them (Earth sphere \( R = 6361450.582 \)m).
Table 8: Angles measured on Bessarabia map between meridians and parallels at the graticule nodes.

<table>
<thead>
<tr>
<th>φ</th>
<th>λ</th>
<th>2&lt;sup&gt;nd&lt;/sup&gt; quadrant</th>
<th>1&lt;sup&gt;st&lt;/sup&gt; quadrant</th>
</tr>
</thead>
<tbody>
<tr>
<td>46°</td>
<td>44°</td>
<td>90°17'</td>
<td>89°43'</td>
</tr>
<tr>
<td></td>
<td>45°</td>
<td>90°53'</td>
<td>89°07'</td>
</tr>
<tr>
<td></td>
<td>46°</td>
<td>88°54'</td>
<td>90°15'</td>
</tr>
<tr>
<td></td>
<td>47°</td>
<td>89°26'</td>
<td>89°57'</td>
</tr>
<tr>
<td></td>
<td>48°</td>
<td>88°56'</td>
<td>89°55'</td>
</tr>
<tr>
<td>47°</td>
<td>45°</td>
<td>90°08'</td>
<td>89°39'</td>
</tr>
<tr>
<td></td>
<td>46°</td>
<td>89°29'</td>
<td>89°44'</td>
</tr>
<tr>
<td></td>
<td>47°</td>
<td>89°59'</td>
<td>89°57'</td>
</tr>
<tr>
<td>48°</td>
<td>44°</td>
<td>90°33'</td>
<td>89°19'</td>
</tr>
<tr>
<td></td>
<td>45°</td>
<td>89°32'</td>
<td>89°32'</td>
</tr>
<tr>
<td></td>
<td>46°</td>
<td>90°00'</td>
<td>89°55'</td>
</tr>
</tbody>
</table>

Results and Discussions

The Russian Empire Map

By analyzing the aspect of the vectorized graticule on the map, it can be assumed that the parallels form arcs of concentric circles and the meridians are straight lines intersecting in the center of the circles, thereby meaning that a normal conic projection was used in the drafting of the map.

Analyzing the values of the angles between the meridians and the parallels measured on the map at the graticule nodes, it was observed that they were not equal to 90°, so we assumed that the used projection was not a conformal one.

Upon examining the differences between the calculated meridian arc lengths and those measured on the map, reduced at the scale of 1:8 300 000, it can be seen that they vary between -0.16 mm and +0.19 mm. Taking into account the fact that the measurement accuracy on a printed map is 0.2-0.3 mm, we can consider the lengths of the meridian arcs equal to those on the surface of the sphere, within the limits of tolerance. Therefore, we can assume that the map was made in an equidistant projection along the meridians.

Regarding the differences between the calculated and measured parallel lengths, it can be seen from Table 2 that they feature values smaller than 0.3 mm on the parallels of latitudes 50°, 55° and 60°, the smallest differences being registered on the parallel of latitude 55°. This way, we can assume that the representation was made on a tangent or secant cone, having as standard parallel that of latitude 55°, this being also the central latitude for the mapped area.

In order to verify these statements, we calculated rectangular coordinates in the Ptolemy and Delisle equidistant conic projections which were used to draw the maps of Russia (Soloviev, 1955). For both hypotheses, the surface of the Earth was approximated to a sphere with radius a of 6362884.283 m (which is the average radius of curvature of the Walbek ellipsoid for latitude 55°). Superimposing the graticule of the Ptolemy projection over the map of Russia, it can be seen that the differences between the nodes of the two graticules vary between 0.9 mm and 14.9 mm (as can be seen in Table 9 below).

Regarding Delisle’s equidistant conic projection, the differences from the graticule on the map of the Russian Empire vary from 0 mm to 39.4 mm. These values are also presented in Table 9.
Table 8: Differences from grid nodes on the Russian Empire map and Ptolemy and Delisle projection.

Analyzing the differences between the node grids on the Empire map and those calculated in the Ptolemy and Delisle equidistant conic projections, it can be observed that the former projection is closer to the projection used for studied map, yet most of them are still much larger than the measurement error, so we cannot say for certain that this was the projection used.
By analyzing the aspect of the vectorized graticule on the map of Bessarabia, it can be assumed that the parallels represent the arcs of circles, whereas the meridians are concurrent straight lines. We assumed that a normal conic projection was used in its drafting.

Analyzing the values of the angles between the meridians and the parallels measured on the map at the graticule nodes, it was observed that they were not equal to 90°, so we assumed that the used projection was not a conformal one.

Upon examining the differences between the calculated meridian arc lengths and those measured on the map, reduced at the scale of 1:1.100.000, it can be seen that they vary between +0.05 mm and +0.18 mm when the Earth surface was approximated to the Walbek ellipsoid, and between +0.03 mm and +0.17 mm in the hypothesis that the Earth is a sphere. Taking into account the
same measurement accuracy on a printed map as in the case of the Russian Empire map, we can consider that the lengths of the meridian arcs are equal to those on the surface of the ellipsoid/sphere, within the limits of tolerance. Therefore, we can assume that the map was made in an equidistant projection along the meridians.

Regarding the differences between the calculated and measured parallel lengths, it can be seen from Table 6 that they vary between -0.2 mm and +0.2 mm in the hypothesis Earth – Walbek ellipsoid, and between -0.25 and +0.14 if the Earth is approximated to a sphere (see Table 7). The smallest differences were registered on the central parallel of latitude 47° in both cases. This way, we can assume that the representation was made on a tangent or secant cone, having as a standard parallel the parallel of latitude 47°.

Although the parallels did not appear to be concentric circles, we calculated rectangular coordinates in the Ptolemy and Delisle equidistant conic projections, as well as in the pseudoconic Bonne projection. For both hypotheses, the surface of the Earth was approximated to that of a sphere with a radius of 6361450.582 m (which is the average curvature radius of the Walbek ellipsoid for latitude 47°). Superimposing the graticule of the Ptolemy projection over the map of Bessarabia, it can be seen that the differences between the nodes of the two graticules vary between 0.0 mm and 4.67 mm. Regarding Delisle’s equidistant conic projection, the differences from the graticule on the map of the Russian Empire vary from 0 mm to 6.87 mm. As pertains to the Bonne projection, the differences are smaller than in the two previous projections, varying from 0 mm to 3.93 mm. All of these values are also presented below in Table 9.

<table>
<thead>
<tr>
<th>Grid node</th>
<th>Distances map – Ptolemy [m]</th>
<th>Distance/n [mm]</th>
<th>Distances map – Delisle [m]</th>
<th>Distance/n [mm]</th>
<th>Distances map – Bonne [m]</th>
<th>Distance/n [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>φ</td>
<td>λ</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>46°</td>
<td>44°</td>
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<td>7555.50</td>
<td>6.87</td>
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<tr>
<td></td>
<td>46°</td>
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<td>1.26</td>
<td>2582.52</td>
<td>2.35</td>
<td>1322.48</td>
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<td></td>
<td>48°</td>
<td>2358.61</td>
<td>2.14</td>
<td>1149.08</td>
<td>1.04</td>
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<tr>
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<td>44°</td>
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<td>2.60</td>
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</tr>
<tr>
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<td>0.00</td>
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<tr>
<td></td>
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<td>0.58</td>
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<tr>
<td>48°</td>
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<td>0.84</td>
<td>2332.15</td>
<td>2.12</td>
<td>1839.25</td>
</tr>
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</table>

Table 9: Differences from grid nodes on the Russian Empire map and Ptolemy, Delisle and Bonne projections.
Figure 3: Grid on the map (black) versus grid in the Ptolemy conic equidistant map projection (green). Map available at web portal https://www.loc.gov/item/2018688650/.

Figure 4: Grid on the map (black) versus grid in the Delisle conic equidistant map projection (red). Map available at web portal https://www.loc.gov/item/2018688650/.
Identifying the projection in which a map has been drawn can be done based on its graticule of meridians and parallels. This task can be easier to perform and more likely to succeed for smaller-scale maps, while being more complicated and not always satisfactory when applied to medium-scale maps. The overall analysis of the shape of the meridian and parallel graticule may suggest the map projection class being used (e.g. cylindrical, azimuthal, conic, etc.). It can also be ascertained whether one of the meridians is represented as a straight line and thus forms the axis of symmetry for the others. In order to identify its projection group, measurements must first be done on the map. Based on the measurements of the lengths of the meridian and parallel arcs, it can be deduced whether some of them are represented without distortions and, therefore, if the projection is equidistant along them. By measuring the angles between the meridians and the parallels, it can be seen whether the projection is conformal or not. The last step is then to determine the actual map projection being used.

In the present paper, these steps outlined above were completed for two maps, one at a small scale (a map of the Russian Empire) and another one at a medium scale (a map of Bessarabia). For the map of the Russian Empire, based on the measurements done on the meridian and parallel graticule, it can be concluded that a normal conic projection, equidistant along the meridians, was used. From this group were chosen two projections used over time to represent the territory of Russia, namely the Ptolemy and the Delisle normal equidistant conic map projections. From the comparisons made, closer results were obtained for the Ptolemy projection. However, since the differences obtained between the nodes of the network drawn on the studied map and those in the Ptolemy projection are significant – though they might be caused by the measurement accuracy on the map, the deformation of the paper and the distortions resulting from its scanning – we cannot say for certain that this is the projection that was used.

Regarding the map of Bessarabia, the problem is even more complicated. We can only say that it belongs to the class of projections representing parallels through circles. Although the distances between
the centers of the circles were found to be large and could not be said that they were concentric, the network on the map was compared to the Ptolemy and Delisle normal conic equidistant projections, since the meridians were found to be undistorted and we assumed that the same projection was used for all of the maps contained in the atlas. Since the measurements of the parallel arcs showed that they were equal to those on the ellipsoid, within the limits of the measurement accuracy on the map, the Bonne pseudoconic projection was also analyzed (Snyder 1997:138).

The graticule in this projection is closer to the one appearing on the map, however the differences remained large enough to prevent stating with certainty that this was the projection used to draw up the map of Bessarabia.

Therefore, identifying the projection of a map based on the network of meridians and parallels constitutes a very complicated and laborious task, and the final result cannot always reveal the actual projection being used, but only the class and possibly the group of projections it belongs to.

References


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-- General Map of the Russian Empire and the Neighboring Polish Empire and Grand Duchy of Finland available at web portal https://www.loc.gov/resource/gdclccn.2018688695/?r=-0.017,0.285,0.15,0.103,0, last accessed in May 2022.