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Georeference of Map of Denmark by Bugge and Wessel (1762-1777)

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Summary: The first systematic map survey covering the Kingdom of Denmark was carried out in the second half of the 18th century, using the experience of the cartographic works by C-F Cassini in France. The head surveyor was Thomas Bugge, who has great contributions from the mathematician Caspar Wessel, the inventor of the complex number plane. 16 sheets of the resulted map product covered the Jylland peninsula and the main Danish isles with a scale of (cca) 1:120000 and another sheet with double scale (1:60000) shows the island of Bornholm. The western part of the series, covering the peninsula, is consisting of 11 sheets of 2 columns, connecting at the meridian west of Copenhagen by 3.2 degrees. These 11 sheets are uniform in extents and connecting the each other at the edges. The other 5 sheets are different in shape and size; however, the scale is the same to each other and to the western sheets. The coordinates of 23 original survey points, published by Bugge, were coupled with their locations in WGS84, and the Cassini (with a center at the Round Tower in Copenhagen; latitude=55.6822 degrees in local datum; 10.2615 degrees east from Paris) projection was tested to provide best fitting results. Parameters of the local datum are: sphere of Picard (R=6372056 m); dX=+11521 m; dY=+1101 m; dZ=–18086 m. Using the results, a georeference method is suggested for the western sheet system, while the original point list can be applied for the eastern ones. The error of georeferenced goes up to cca 700 meters at less accurate regions.

Introduction

Over the last decade, an explosion of publicly available data sources has facilitated research into the history of maps and geodesy. A large number of scanned historical maps have become available, in many cases on the websites of national geodetic services. In addition, the Google Books service has made available the full range of books published before 1900, as well as the substantial part of periodicals that can be catalogued as books. This provides an excellent starting point for georeferencing the historical maps that are now available: matching them to modern coordinate systems. In this way, they can be used as an overlay for today's maps, as a source of geospatial data.

The easiest way to match old maps to modern ones used to be to use the same georeferenced points (GCPs) that are recognizable on both old and modern maps, assuming some mathematical relationship between the two coordinate systems. However, this method is only of limited use in the case of multi-sheet map works where, in addition to the accuracy of the individual sheet alignment, the precise connection of the sheet boundaries had to be solved. Although the latter is mathematically solved (Molnár, 2008), there is currently no GIS tool (e.g. QGIS extension) available to implement it in practice. The standard solution for georeferencing map works consisting of several sheets is to know the geodetic datum of the old map, the map projection (or its approximate model) and to place the sheets in this coordinate system.

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The first topographic mapping of Denmark was carried out in the second half of the 18th century, in cca. 1:120000 scale (Borre, 2014), using the methodology of the first major mapping project in Europe, the Cassini survey of France, at the same time as, or slightly before, the first military survey of the Habsburg Empire. The creators of the map work from 1762 to 1777 were Thomas Bugge and Caspar Wessel; the latter's first major work in his scientific career (Kristensen, 2001). Wessel later improved his survey methods and partly from this experience made a name for himself in mathematics as the pioneer of the complex number plane (Branner & Johansen, 1999).

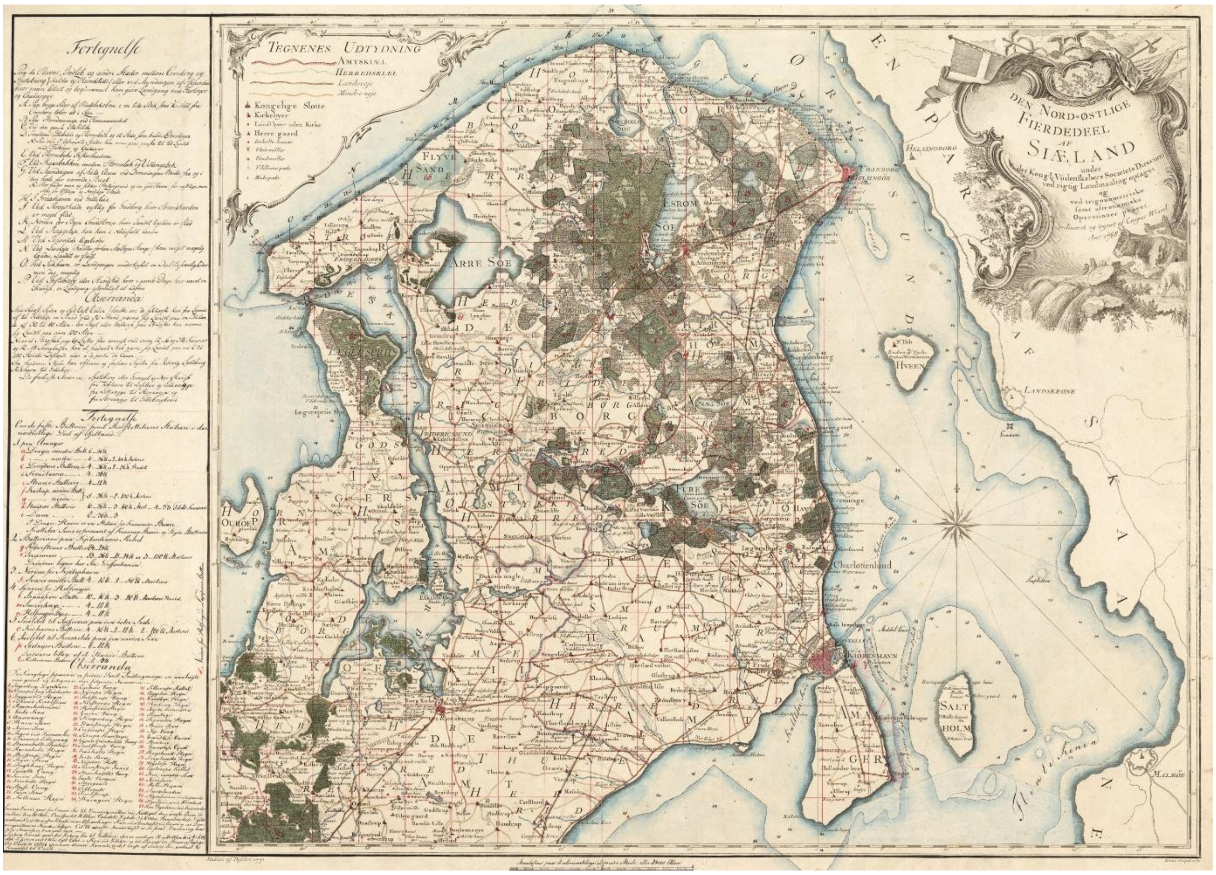

Figure 1. The sheet 14, showing the area of Zealand. This version shows the red projection grid lines according to the Copenhagen-based Cassini projection.

In recent years, the Danish Geodata Agency has made available a wide range of historical Danish cartographic data sources, including the Bugge-Wessel map series. In this paper, I will use contemporary point lists and other metadata from original works available in Google Books to provide the coordinate system and geodetic datum of the map work for use in GIS software, and to demonstrate the accuracy of the georeferencing.

Projection and geodetic basis of the map work

As the French Cassini survey was used as the model for the map, it is not surprising that the projection of the map is also the Cassini projection. This obvious assumption is supported by the structure of the point lists published in Thomas Bugge's books (in the Appendix of Bugge, 1779; Bugge, 1787, pp. 132-134; Figure 2). The original coordinates of the geodetic base points are given in 'Meridian' and 'Perpendicular' units, similar to the Cassini survey (Timár et al., 2014). The unit of length used was the Danish ‘ale’, whose length can be estimated at 0.629 m;). Knowing the projection type alone is not sufficient for georeferencing, the projection center must also be known.

Tabelle

über die Lage der trigonometrischen Stationen und anderer Oerter gegen die Mittagslinie des Kopenhagener Observatoriums und ihrer Perpendikular.

Namen der Stationen und Oerter.	Abstand von der Kopenhag. Mittagslinie nach Dänischen Ellen.	Abstand von der Perpendikular zur Mittagslinie nach Dän. Ellen.
Trigonometrische Stationen in Seeland.		
Bröndbyehügel bey Bröndbye gegen Morgen	14041 westl.	3911 südl.
Gerichtshügel bey Mörkhügel	8582 —	9539 nordl.
Bovnehügel bey Ballerup	21001 —	8648 —
Signal bey Oelstykke	39776 —	18227 —
Steensknol bey Hiorlunde	38394 —	26021 —
Maglehügel bey Ude-Sundbye	49689 —	27125 —
Der Berg bey Ströe	42050 —	35884 —
Maglehügel bey Brederöd und Friedrichswerk	53518 —	50548 —
Die Schanze bey Friedrichsburg	23899 —	44847 —
Fröebakken auf Flyve-Sand	48434 —	65593 —
Kirche zu Dyssen bey Blidstrup	36859 —	71767 —
Lundebakken bey Söeburg	23512 —	71590 —
Salgaard oder Veibye-Klint	44882 —	71350 —
Oelhügel bey Smidstrup Strand	35253 —	76580 —
Bovnehügel bey Gilleleie	23891 —	77824 —
Signal bey Apperup	4653 —	70879 —
Spodsberg bey Kiffavn	71349 —	52457 —
Skreberg oder Nakkehoved, in Ods Gerichtsbarkeit	82316 —	42285 —
Hvedshügel bey Hvedstrup	38782 —	2363 —
Roeskilder Heidemühle	47211 —	7313 südl.
Sälsöer-Mühle	59020 —	11819 nordl.
Elstallebierg	70663 —	10882 südl.
Bovnehügel bey der Kirche zu Syv	53469 —	18974 —
Blakker-Mühle in Horns Gerichtsbarkeit	66887 —	15806 nordl.
Mörkemose-Berg	89649 —	7595 südl.
Knösen bey Skamstrup	110561 —	10700 —
Vairhügel bey Draxholm	117927 —	20838 nordl.
Egebergs-Mühle in Ods Gerichtsbarkeit	89538 —	31000 —

Figure 2. Geodetic base points, surveyed by Bugge and Wessel in Zeaeland. Coordinates are in Danish 'ale' units (cca. 62.9 cm). As Copenhagen is at the eastern edge of the area, most of "Meridionals" are western ones (Bugge, 1787, page 132).

As in the French survey, the center was defined at the main astronomic observatory established in the mapped area. Denmark, with its Tycho Brahe, already had a long tradition of astronomy, however its Uranienborg observatory, used two centuries earlier, was in ruins. At the end of the 18th century, the observatory in the Round Tower (Rundetaarn) in Copenhagen became the starting point for the geodetic network and thus for the mapping of Zealand. The latitude of the observatory is given by Bugge (1787); the longitude is given on page 129 of the cited work as the time difference with the Paris observatory (41 minutes, 2.75 seconds, thus 10.26146 degrees). Adding the known modern Greenwich-Paris difference, the longitude of the fundamental point at the local datum is 12.59867 degrees. This in itself is of little significance for local Danish contemporary cartography, where the meridian at Copenhagen was simply taken to be zero longitude. For us, however, when fitting it to global maps, it is very necessary, and fortunately such 'alignment' of the stars was not unprecedented at this time (Cassini, 1765). The latitude was given as the angle between the celestial pole and the horizon, averaged over a very large number of measurements, as 55 degrees 40 minutes 56 seconds (cited work, p. 124). These two coordinates give the center of the Cassini projection used.

According to Kristensen (2001), the radius of the sphere-shaped Earth, was used as 6372056 metres following Picard (1671), as used by Cassini & Maraldi (1744). Using this, and the coordinates of the Rundetaarn in WGS84, the datum can be parameterized with dX=+11521 m; dY=+1101 m; dZ=-18086 m, which can be used in GIS software. The Zealand part can be georeferenced more accurately than other country maps from this period (cf. e.g. the Norwegian Kvadratmil maps; Timár et al., 2017) using GCPs generated according to the coordinates reported on pages 132-134 of Bugge (1787; Figure 3).

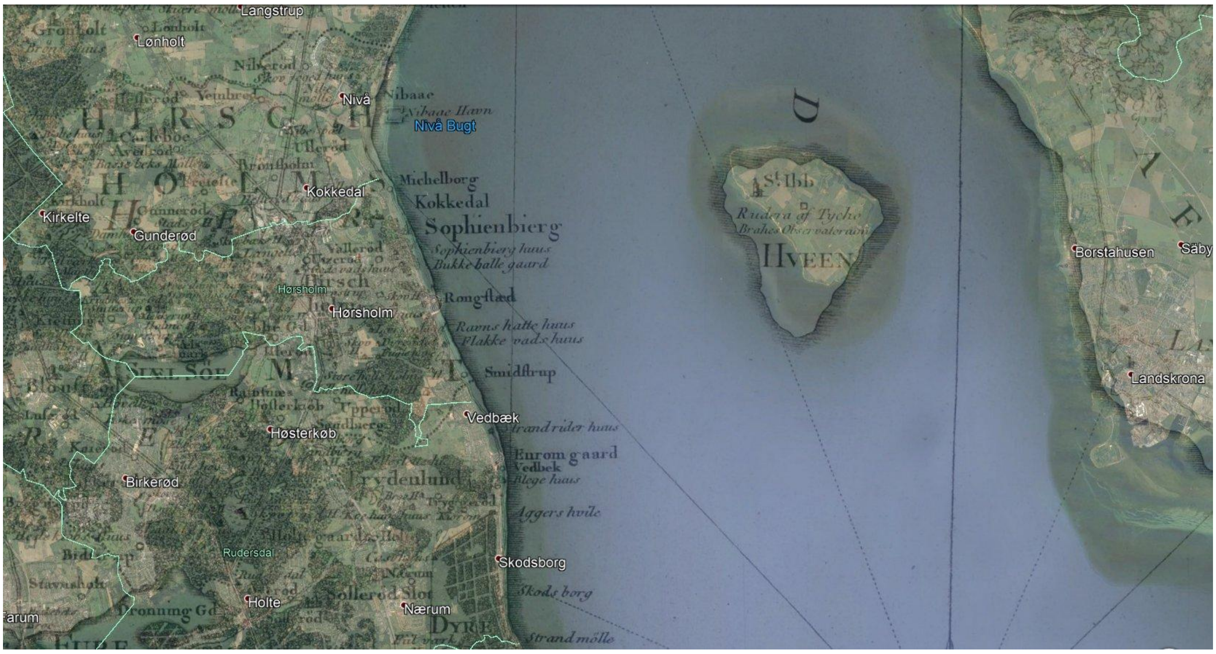

Figure 3. Fit of the Zealand sheet (part) to Google Earth. Fitting accuracy of Danish mainland is quite good, however at the Swedish side, the shoreline is inaccurate. In the middle, there is the Hveen (Ven) island with the indication of ruins of Uranienborg.

The complete sheet system

Zealand, the area around Copenhagen, played the role of a pilot project for the map work. From 1778, the Jutland peninsula and the other islands of Denmark were also mapped. Alongside Caspar Wessel, Ole Christopher Wessel and Niels Morville played leading roles in the work (Borre, 2014). The group of sections depicting Jutland is divided into two columns by a meridian arc running north-south at a distance of 3 degrees 12 minutes west of Copenhagen. Due to the transformation proposed by Bugge, the georeferencing of the sections cannot be solved using the Cassini projection model (Kristensen, 2001). However, with a Cassini projection defined by a midpoint of $\phi 0=55.6314$ degrees; $\lambda 0=9.3758$ (the same as 3 degrees 12 west of Copenhagen), using the geodetic datum given in the previous section, georeferencing of similar accuracy to other maps of the period, although weaker than accuracy at Zealand, can be achieved (Figure 4).

Figure 4. Fitting accuracy at western shore of Jylland (sheet 7).

Acknowledgement

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