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Preliminary numerical investigations on the “Liber de Existencia Riveriarum et Forma Maris Nostri Mediterranei”

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Summary: The discussion about the origin and the evolution of the portolan charts, more than seven centuries after the appearance of the oldest extant the Carta Pisana, is still lively on going. Despite the advances made in the recent years both in the historical – thematic and the geometric aspects of this map typology there is still open and inviting the research especially in the view of the textual data appeared in the meantime. The issue of the geometrical properties of the portolan charts is a field of great interest deserving advanced research not only on the map content but also on texts related to historical evidence and to geometric and numerical entities relevant to the cartographic features depicted on the charts. Since the Carta Pisana dates from almost the last quarter of thirteenth century any relevant texts and written sources from that period or earlier is worth studying and analysing, especially when it provides numerical values and quantitative descriptions about the lengths and directions governing the important “geometrical fabric” of the portolan chart. A text of such value deserving major interest is the Liber de Existencia Riveriarum et Forma Maris Nostri Mediterranei unveiled in the middle ‘90s by Patrick Gautier Dalché. The strong point in dealing with the numerical analysis of Liber is the fact that the text, written in the second half of twelfth century, almost a century earlier than the Carta Pisana. In this paper a first approximation of Liber’s numerical content is attempted and a comparative analysis is carried out concerning the base line lengths as given in the text with respect to the corresponding lengths as defined today. Following a typical data acquisition and elaboration procedure, known in modern numerical spatial analytics and statistics, a Gis system has been implemented in order to manage and analyse the relevant metrical and geographical data, also in the context of their temporal evolution. Particular attention has been given to the issue of the determination of the conversion ratio of the nautical mile used in Liber into today’s kilometer, a fundamental requirement for any further comparative study of the base line length given in Liber, with respect to today’s counterpart lengths.

Introduction

In 1995 Patrick Gautier Dalché, published his important study on Liber de Existencia Riveriarum et Forma Maris Nostri Mediterranei, a textual guide for navigation in the High Middle Ages Mediterranean Sea, written by an anonymous author from Pisa between the years 1160 and 1200 (Gautier Dalché, 1995). In this noticeable work Gautier Dalché does not hesitate to avoid rigor by referring to “many indications … that lead to the belief that…” ¹ for aspects related to some so far unanswered questions about the origin and evolution of portolan charts. Liber is dated almost a century before the Carta Pisana, the earliest extant portolan chart (dated ca 1270) representing the

¹ In French: “nombreux indices… conduisent à penser…” With this statement Gautier Dalché postulates the legitimacy of the logical reasoning based on relevant indications and not only on evidence (i.e. proofs), especially when evidence is missing.
Mediterranean coastline in the small scale of approximately 1:4,000,000 (i.e. 1 cm on Carta Pisana corresponds to 40 km on sea surface).

Despite the announcement of Liber’s author about a map to be added in the descriptive text, no such map is associated to Liber so far and the extant text is devoid of maps, providing only navigational guiding and numerical geometric data concerning base lines between coastal sites, namely length-distances in miles and oriented angular directions relevant to some (mental) wind roses.

Following the still on going and lively discussion among the experts of portolan charts cartography, especially on their origin and evolution, the challenge was great to attempt some first numerical comparative elaborations of the relevant data given in Liber. The idea was based on a simple and straightforward approach, making full use of the available digital methods and techniques known since long time in cartography relevant to the geometric content of maps (e.g. Guerra et al., 1999; Livieratos, 2000; Balletti, 2001; Livieratos, 2006).

The aim of this preliminary investigation on the Liber’s numerical data was to have a first evaluation about the reliability or the order of convergence of the magnitudes of the two fundamental elements of early portolan charts; i.e. the base line lengths and angular directions.

The collateral requirement for this was the study of the proper identification and the actual spatial placement of the toponyms given in the text and the magnitude of the medieval nautical mile, both in terms of the today’s counterparts.

Only after the analysis and the arrangement of these two fundamental items, which both refer to a “conversion issue” of the place names and the units of length, it would be possible to conclude with confidence on the identification of the proper base lines to use in a next comparison scheme and on the validity of the differences between the lengths used in the High Middle Ages and those of today, associating the relevant uncertainty figures to the analysed values.

An important addendum in Gautier Dalché’s publication about Liber is the preparation of two supporting modern maps on which are represented some of the base lines, with assigned toponyms, reported in Liber. The first map, called here A, represents the Italian peninsula and the Aegean Archipelago and the second one, B, represents the entire Mediterranean Sea.

This helpful depiction gives the grounds to develop a numerical and cartographic analysis and management of data given in Liber and assists their transfer into a Gis environment for further elaboration. Both supporting maps A and B was used in our study for the visualisation of the processing and the additional representation of the results obtained.

Study and placement of toponyms

The first step of our study was to recognize and localise the toponyms named in Liber. This was necessary in order to proceed to the identification of the sites, mainly coastal, with respect to their today’s toponyms and their placement on a modern map georeferenced with proper coordinates. Needless to say how difficult and demanding it is the spatial identification of old toponyms with respect to their modern counterparts, to which most of the times have assigned different name. This part of the work was the most sensitive but at the same time a challenging part approached by a constructive interdisciplinary cooperation.

The second step was to introduce properly the maps A and B into a Gis environment combined with the Liber data, namely the sites on the coastline (mainly ports) with the associated toponyms and

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2 The conversion of lengths in sea was a subject of constant dispute since the antiquity, as Strabo critically commented all the authors dealing with geography who disagreed on the length of distances (cf. Strabo: VI, 3, 10 = C 285; Medas, 2008).
the base lines interconnecting the sites, in order to have all data in a common computational and visualization embodiment.

The third step was the georeferencing using a standard Gis application (Balletti, 2006). The two supporting maps A and B, apparently given in conic projection, allow the use of the Lambert conic conformal projection with central meridian at 15 degrees eastwards. For georeferencing, the coordinate fitting transformation process, selected among others obeying specific properties (e.g. Boutoura & Livieratos, 2006) is relevant to a global second order polynomial function, using for the fitting 52 control (common) points recognized both in the Liber data set and the modern counterpart set, concerning map A and 29 control points for map B.

Figure 1. The georeference of the two maps A (up) and B (down) with the distribution of control points. The supporting maps are redrawn in a Gis environment from Gautier Dalché (1995)

3 In global transformations, the same parameters are valid for every point of the image, once the model has been chosen; they are the traditional planar transformations that relate a system of points to another set of points in a one-to-one correspondence.
The procedure implemented here allows the easier placement of toponyms in their own geographical area for obtaining a reference basis due to which could be possible the localising of ports with higher closeness. Following this procedure the point discrepancies obtained after the fitting transformation were rather low, 2 km for map A and 11 km for map B. These deviations are not as relevant as in the normal georeferencing usually applied in mainstream cartography. As a result, all the toponyms are placed not only by georeferencing properly the two maps, but also after some inquiry carried out on historical names of sites considered in the procedure. Starting from the general positioning of the ports in Liber, thanks to the georeferentiation of map A and map B, we were able to identify 124 base lines and 158 toponyms, 138 of which were identified in Ptolemy’s Geographia name and point positioning list, a not at all easy task due to the alterations in the names of common places occurred in the course of time, often severe.

Figure 2. The Mediterranean Sea with the depicted base lines defined in Liber. The supporting map from Gautier Dalché (1995)

The output of this preliminary analysis in three steps, using the two supporting maps prepared for Gautier Dalché’s publication, is the listing of all the base lines reported in Liber in miles and represented in maps A and B; to these values some other information are correspondingly assigned, extracted from the Liber text commentary by Gautier Dalché, e.g., about the orientation of angular directions given in Liber, about the Carta Pisana and about modern maps; the toponyms in Compasso da Navigare, and the mile distances of base lines in Compasso. When applicable, these data are also properly associated with corresponding toponymy and regional reference, extracted from Ptolemy’s Geographia, with assigned Ptolemaean coordinates of latitude and longitude given in properly elaborated recent catalogues (Tsorlini, 2011).

A Gis system for the data archiving

Working with heterogeneous data coming from different sources of different time periods it is always necessary and efficient to implement an information management tool able to relate data together. Dealing with spatial data, as it is the case of this study, a Gis system has been used in order to manage and analyse metrical and thematic data in their historic-temporal evolution, helping thus to put historical information under a spatial interpretive key.
The potential capability of Gis as an effective tool for managing and archiving heterogeneous data is already well known (e.g. Gregory & Healey, 2007; Rinaudo et al., 2007; Barzaghi et al., 2012; Carrion et al., 2013; Gatta & Bitelli, 2016). In this research a proper database was created, available for a variety of spatial analyses with geographical reference including enquiries about the entities embedded. Several attribute tables developed within the database linked with the toponyms and the base lines under processing.

At the beginning of this work we therefore had to design a table structure where to catalogue all the data extracted from the analysis of the base lines given in Liber starting from toponymy. Once the tables were completed with the data obtained from the sources, we created a relational database: in fact, through this tool we can carry out queries with complex syntax, select information according to various criteria, consult the archive in an easy and efficient way and, at the end, obtain representations of data collected in a defined cartographic reference system.

Inside the used Gis platform (here the ArcGIS) we drew all the points along the coastline related to the toponyms identified in Liber; through join operations we linked these entities to a table with the collected information and visualised in a spatially referenced system the toponymy of the places in three historical periods to be possibly studied (e.g. modern, Liber, Ptolemy’s Geographia).

![Figure 3. The Gis system associated with the database carrying historical information.](image_url)

After the identification of toponyms, we drew the base lines connecting couples of points. Then, these segments were linked to the table containing the information on Liber’s base line distances and orientations, other data offered by Gautier Dalché’s authorship, the information about the relevant Ptolemaean coordinates and their regional reference in Geographia.

**The conversion factor of medieval nautical miles**

After the collection and cataloguing of data, all sets of values analysed in detail, namely the Liber base line distances (in nautical miles), the modern counterparts (in km) and those in Ptolemy’s Geographia (in km). The modern base line distances obtained from the processing within the Gis enabling the necessary calculations and results about the geodesics of the base lines, given the pairs
of coordinates at both ends of the line. The relevant values of Geographia base line distances derived from the Ptolemaean geographical coordinates, applying the haversine formula, which determines the important in navigation great-circle distance between two points on a sphere given their longitudes and latitudes.

The interesting issue at this point is the relation between the Liber values given in nautical miles (nm) and the kilometres (km) computed in the Gis environment. This relation is important for the conversion of the base line values of lengths from the sake of the comparative analyses and the interpretative models to treat, taking into account the heterogeneous data and information given in historical texts like Liber. The available literature on the issue does not offer univocal answers on the conversion problem of the nm to km correspondence in the High Middle Ages, the Liber was written; nevertheless, the majority of authors converge at the ratio 5:6 for the nautical mile to the land mile, which means a correspondence of 1 nm to 1.233 km.

In this study, in order to decide on the conversion ratio km/nm, we analysed the ratio km/Liber’s nm (km/Lnm) as derives from the relevant testing of the base lines given in Liber. The analysis of the statistical distribution of this ratio shows that the result reads around the average value of 1.34 km/nm with median value 1.17 km/nm.

The visualisation of the spatial distribution of the km/Lnm ratio derived from the analysis of the base lines was elaborated using the Gis application. Each base line carries its own conversion ratio and all values grouped in three classes according to their order of magnitude, for easy distinction of the value of ratio associated to each base line.

In Fig. 4 it is shown that 36 base lines, in purple, carry the ratio band $0.45 \leq \frac{km}{Lnm} \leq 1.09$; 28 base lines, in red, carry the ratio band $1.10 \leq \frac{km}{Lnm} \leq 1.39$ and 27 base lines, in black, carry the ratio band $1.40 \leq \frac{km}{Lnm} \leq 4.37$.

![Figure 4. The classification of base lines in Liber according to the values of the length conversion ratio km/Lnm. The supporting from Gautier Dalché (1995)](image)

From these first analyses of the conversion ratio, km/Lnm, we could not deduce any particular pattern of concentration of values in some specific areas of the Mediterranean Sea; this could be done after further statistical analyses and a proper data classification.
A statistical approach for the analysed base lines

Since the very first numerical elaboration of the data given in Liber, we noticed different factors affecting the identification of the uncertainties in the analysed data, majorly the one concerning the degree of rounding (approximation) of the values of base line lengths and orientation. The analysis showed that the rounding of the long (200 to 700 \( L \)nm) base lines is of the order of 50 \( L \)nm and the rounding of the shorter base lines reaches 10 \( L \)nm.

This clear approximation needs to group the data into two main classes, one concerning the short and medium distances and the other concerning the longer base line lengths. In addition to that, the shorter base lines are treated in a different way, due to the fact that in many cases it is difficult to precisely identify the today’s location of the sites of ports as named in Liber, for which even a slight deviation in the placement of the today’s location would have introduced a high relative uncertainty in the process.

Starting from this, the base lines we grouped the base line in three classes (Fig. 5):
- Class 1: short base lines the length ranging between 10 and 50 \( L \)nm
- Class 2: medium base lines the length ranging between 60 and 160 \( L \)nm
- Class 3: long base lines the length ranging between 200 and 700 \( L \)nm

For the classification we followed a reasoning based on the careful analysis of data. The threshold used to divide into two classes the previously one of the short and medium base line lengths was identified calculating the average of the ratio \( \text{km/Lnm} \) for every possible value of the threshold, choosing the one that would have allowed a better distinction of two different base line groups.

In fact, observing the results of this analysis, it is possible to notice a gap between the 50 and 60 \( L \)nm base lines: the average ratio calculated for all the base lines to 50 \( L \)nm is 1.59 \( \text{km/Lnm} \), while, moving the threshold to the following step, namely the lengths up to 60 \( L \)nm, the ratio average reads much different, i.e. 1.39 \( \text{km/Lnm} \). Instead, if we examine the average including into the data set the 70 \( L \)nm base lines, we can see that there is not such a high gap as before; in this case the average ratio is 1.37 \( \text{km/nm} \), very close to the one concerning the 60 \( L \)nm base line group.

![Figure 5. Histograms of the distribution of base line lengths given in Liber divided into the three classes of length.](image)

An attempt to distinguish between the different factors affecting the uncertainty of the analysed base line length is based on the definition of a proper general model formulated by the hypothesis-model equation:

\[
S_L = S_a \left( r_L + m_1 + m_2 + m_3 \right)
\]
where:
$S_L$ is the base line length in *Liber*
$S_a$ is the today’s (actual) base length counterpart
$r_{L}$ is the ratio $\text{km}/\text{Lnm}$
$m_1$ is the uncertainty of today’s values in relation to short base line lengths; in this case the uncertainty derived from the identification of today’s sites of ports, counterparts of those in *Liber*, plays a significant role
$m_2$ is the uncertainty of base lines in *Liber*
$m_3$ is the degree of approximation identified in the base lines reported in *Liber*

For each of three Classes, the above model-hypothesis equation takes the forms:

Class 1: $S_L = S_a (r_L + m_1 + m_2)$
Class 2: $S_L = S_a (r_L + m_2)$
Class 3: $S_L = S_a (r_L + m_2 + m_3)$

In order to estimate the conversion ratio and the error, it has been applied a linear regression analysis for each class. We started from the less complex set of cases, the Class 2, as the factors $m_1$ and $m_3$ relate solely to the two Classes 1 and 3. The regression line shows a positive relation between the two distance measures which is highly significant from a statistical point of view: the coefficient of determination $R^2$, indicating the proportion of the variance in the dependent variable that is predictable from the independent variable, used in statistical models, like the one used here, targeted at the prediction of future outcomes or the testing of hypotheses, on the basis of other related information, providing a measure of how well observed outcomes are replicated by the model, based on the proportion of total variation of outcomes explained by the model (Draper & Smith, 1998), is for the Case 2 model $R^2 = 0.7$, close to the maximum value 1.0, suggesting thus the ratio $\text{km}/\text{Lnm}$ equal to $1.3 \text{ km}/\text{Lnm}$.

Even more interesting are the results from the analysis of the regression for the Class 3: the angular coefficient of the regression line is 1.2 (very close to the km/nm conversion ratio suggested in the scientific literature related to portolan charts) and the function is far more significant from a statistical point of view, as it helps explaining $85\%$ of the relative variability of the measures ($R^2 = 0.85$).

The linear regression analysis offered less significant results for Class 1, concerning the short base lines, since the determination coefficient is low $R^2 = 0.38$; in this case, the uncertainty factors are very high and it is difficult both to identify and to interpret because a small uncertainty e.g. in the spatial identification of the site of ports leads to high differences between the today’s short base line lengths and the corresponding counterparts given in *Liber*. 

[22]
Neglecting thus Class 1, the statistical analysis represented by the coefficient of the regression line suggests a base line length conversion ratio equal to 1.3 km/Lnm for Class 2, and 1.2 km/Lnm for Class 3. Since the conversion rate of Class 3 is very close to the one already given in the relevant literature, we opted to use both, the 1.3 km/Lnm ratio estimated here and that suggested in literature 1.233 km/nm.

Testing a conversion ratio $1.23 \leq \frac{\text{km}}{\text{Lnm}} \leq 1.3$, we can estimate the difference between the two corresponding sets of base line lengths, the today’s and those of Liber’s.

The difference is represented by the relative error after the comparison of the today’s and Liber’s base line lengths, after the grouping in the three Classes, rounded to 10 nm for the lengths up to 160 nm, and to 50 nm for the longer lengths, keeping in mind the shortcomings associated to the short base lines. This testing allows to conclude which one of the two conversion ratios leads to the smaller relative error, allowing to suggest the most probable conversion factor for the High Middle Ages nautical mile. The analysis of the relative error is not presented here, but it is anticipated the result obtained from our research: the two statistical analyses performed, showed that the relative errors are always lower using the ratio value 1.233 km/nm instead of the 1.3 km/nm value.

Part of this research was also to inquiry on the possible relevance, if any, of Ptolemaean base lines, as derived from Geographia, corresponding to the Liber’s counterparts. The first results of this study show an overall weak relevance, taking into account the difficulties of the proper identification of the toponymy correspondence, the reliability of the Ptolemaean coordinates from which the base line lengths are computed and a number of other analytical and modelling problems that should be further analysed.

On the other hand, this first approach showed that in some limited cases very long base lines given in Liber look somehow close in length to those computed from the Ptolemy’s coordinates (cf. Livieratos, 2006), an issue meriting closer and systematic investigation.

**Conclusions**

This preliminary study aims at the formulation of a method for the analysis of complex and heterogeneous spatial data testing possible approaches for the study of the texts referred to the early portolan charts as it is the exceptional example the Liber de Existencia Rivetarum et Forma Maris Nostri Mediterranei presented in 1995 by Patrick Gautier Dalché. The study focuses entirely on Liber treating statistically the question of the nautical mile conversion and opening the perspective for extensions in relevant fields on portolan charts research, concerning the geometrical content of this medieval map typology, including the issues related to map projections (Boutoura, 2000; Balletti & Boutoura, 2001).
It is an experimental technical study on an important historical text from the High Middle Ages, masterly annotated by a prominent scholar of medieval texts, without dealing here with issues related to the origin and evolution of the portolan charts. It is focused on the displaying of how modern digital systems for spatial analysis, as it is the Gis technology, could be successfully implemented in the scientific investigation of the textual sources concerning the portolan charts, in terms of statistics.

Following the Gis accumulated experience in collecting, evaluating, analysing and visually displaying the geometric type of heterogeneous data given in the texts, in terms of modern spatial statistics, comparative analyses can follow, e.g. on spatial map deformation (Boutoura & Livieratos, 1986; Livieratos, 2006a) involving the early nautical charts e.g. the Carta Pisana (Fig. 7), on which 1 mm linear resolution on the map corresponds to approximate 4 km on the sea, or 1 cm to 40 km correspondingly, due to the map scale and maps relevant to the textual content (Livieratos, 2017).

The preliminary results obtained in this study prepare the terrain for further advancing the research on the portolan charts semantics, especially referred to the critical period of the twelfth and thirteenth centuries, keeping always in mind an inspiring key point by Gautier Dalché, in his publication about Liber, stating “…as early as the middle of the twelfth century, there were collections of nautical instructions, or portulans, the realization of which was based on the secular experience of seamen”.  

Acknowledgements

The cooperation in Cartographic Heritage of the two groups in Thessaloniki and Venice started in the late ‘90s, (see, e.g., Guerra et al., 1999; Livieratos, ed., 2000). Today it is advancing in research.

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4 In French: “…il existait, dès le milieu du XIIe siècle, des recueils d’instructions nautiques, ou des portulans dont la réalisation s’effectua à partir de l’expérience séculaire des marins.”
linking the University Iuav of Venice Laboratories of Cartography & GIS and of Photogrammetry with the Aristotle University of Thessaloniki Laboratory of Cartography & Geographical Analysis.

References


